Part 2 of 5:

Free Riding and Vote Management under Proportional Representation by the Single Transferable Vote

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Abstract. We give an overview over free riding strategies and vote management strategies under proportional representation by the single transferable vote (STV). A *free rider* is a voter who does not waste his vote by voting for a candidate who is certain to be elected even without one's vote. *Vote management* is a strategy where a party maximizes its number of seats by asking its supporters to vote preferably for those of its candidates who are less assured of election. We demonstrate that these strategies are a common feature of everyday political life wherever STV methods are being used. Furthermore, we demonstrate that there are mainly only two types of free riding strategies, but a very large and colorful family of vote management strategies. We will introduce a mathematical model to describe free riding and vote management and we will introduce an STV method that is vulnerable to these strategies only in those situations in with otherwise Droop proportionality would have to be violated.

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This paper is the second part of a series of papers that can be downloaded here:

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1. Introduction

When STV methods are criticized, then they are usually criticized for violating monotonicity, participation or consistency (Brams, 1984; Doron, 1977; Dummett, 1984, 1997; Nurmi, 1997; Saari, 1994). However, we are not aware of any instance where a violation of these criteria has actually been misused for strategic purposes. In this paper, we demonstrate that rather free riding (section 3) and vote management (section 4) are the two most serious problems of STV methods.

A *free rider* is a voter who misuses the fact that in multi-winner elections it is a useful strategy not to vote for a candidate who will be elected even without one’s vote. Free riders are frequently used as an explanation why in multi-winner elections popular candidates get fewer votes resp. fewer first preferences than their popularity suggests (Hatton, 1920). But, to the best of our knowledge, there is not yet any empirical study about whether free riding strategies are really causal for this observation and about how many voters actually use free riding strategies. However in section 3 of this paper, we use the ballot data of the City Council and School Committee elections in Cambridge, Massachusetts, to determine the number of voters who use a free riding strategy that has been predicted e.g. by Woodall (1983) and Tideman (2000) (*Woodall free riding*). We do not find any evidence at all that voters use this free riding strategy. We give explanations why voters do not use this strategy (Hylland free riding; possible backfire of Woodall free riding; Woodall cannot be misused by political parties on a larger scale; etc.).

Vote management is a strategy where a party or a group of independent candidates maximizes its number of seats by spreading its votes evenly among its candidates. It has already been predicted by Droop (1881) that vote management is a useful strategy under STV with the Andrae-Hare quota. The term “vote management” is used in this manner since about 1987 (Mair, 1987a, 1987b). Since the mid-1990s, it is accepted that vote management is a useful strategy also under STV with the Droop quota (Gallagher, 1993b, 2003; Marsh, 1999).

We will introduce a theoretical concept to describe vote management (section 6.1). This concept will be used to introduce an STV method that is vulnerable to free riding and vote management only in those cases in which otherwise Droop proportionality would have to be violated (section 6.2). Furthermore, this concept will be used to introduce a method to produce party lists (section 7).

2. Basic Definitions

We presume that A is a finite and non-empty set of candidates. $C \in \mathbb{N}$ with $1 < C < \infty$ is the number of candidates in A.

A binary relation $\succ$ on A is *asymmetric* if it has the following property:

$\forall\ a,b \in A$, exactly one of the following three statements is valid:

1. $a \succ b$.
2. $b \succ a$.
3. $a \approx b$ (where “$a \approx b$” means “neither $a \succ b$ nor $b \succ a$”).

A binary relation $\succ$ on A is *irreflexive* if it has the following property:

$\forall\ a \in A$: $a \approx a$.

A binary relation $\succ$ on A is *transitive* if it has the following property:

$\forall\ a,b,c \in A$: ($a \succ b$ and $b \succ c \Rightarrow a \succ c$).

A binary relation $\succ$ on A is *negatively transitive* if it has the following property (where “$a \succsim b$” means “not $b \succ a$”):

$\forall\ a,b,c \in A$: ($a \succsim b$ and $b \succsim c \Rightarrow a \succsim c$).

A binary relation $\succ$ on A is *linear* if it has the following property:

$\forall\ a,b \in A$: ($b \in A \setminus \{a\} \Rightarrow a \succ b$ or $b \succ a$).

A *strict partial order* is an asymmetric, irreflexive, and transitive relation. A *strict weak order* is a strict partial order that is also negatively transitive. A *linear order* is a strict weak order that is also linear.

A *profile* is a finite and non-empty list of strict weak orders each on A. $N \in \mathbb{N}$ with $0 < N < \infty$ is the number of strict weak orders in $V := \{ \succ_1, ..., \succ_N \}$. These strict weak orders will sometimes be called “voters” or “ballots”.

“$a \succ_v b$” means “voter $v \in V$ strictly prefers candidate $a \in A$ to candidate b”. “$a \approx_v b$” means “voter $v \in V$ is indifferent between candidate a and candidate b”. “$a \succsim_v b$” means “$a \succ_v b$ or $a \approx_v b$”.

A possible implementation of the Schulze method looks as follows:

> Each voter gets a complete list of all candidates and ranks these candidates in order of preference. The individual voter may give the same preference to more than one candidate and he may keep candidates unranked. When a given voter does not rank all candidates, then this means (1) that this voter strictly prefers all ranked candidates to all not ranked candidates and (2) that this voter is indifferent between all not ranked candidates.

$M \in \mathbb{N}$ with $0 < M < C$ is the number of seats.

A_M is the set of the $(C!)/((M!)\cdot((C-M)!))$ possible ways to choose M different candidates from the set A. The elements of A_M are indicated with *wedding* letters $\mathfrak{A}$, $\mathfrak{B}$, $\mathfrak{C}$, ...

$D := N/(M+1)$ is the so-called *Droop quota*.

$H := N/M$ is the so-called *Andrae-Hare quota*.

Suppose there is a set of candidates $\emptyset \neq B \subsetneq A$ such that more than $s \in \mathbb{N}$ Droop quotas of voters prefer each candidate $b \in B$ to each candidate $a \notin B$. Then *Droop proportionality* says that at least min { s, $|B|$ } candidates of this set must be elected, where $|B|$ is the number of candidates in B.

Suppose there is a set of candidates $\emptyset \neq B \subsetneq A$ such that at least $s \in \mathbb{N}$ Andrae-Hare quotas of voters prefer each candidate $b \in B$ to each candidate $a \notin B$. Then *Andrae-Hare proportionality* says that at least min { s, $|B|$ } candidates of this set must be elected.

Droop proportionality implies Andrae-Hare proportionality.

Proportional representation by the single transferable vote (STV) is an election method to fill M seats with the following properties:

(1) Input of this method is a profile V.

(2) Output of this method is a set $\emptyset \neq \mathcal{S}_M \subseteq A_M$ of sets each of M candidates with the following property: If $B_M \in A_M$ doesn't satisfy Droop proportionality, then $B_M \notin \mathcal{S}_M$.

3. Free Riding

The fact, that more and more communities, that use STV methods, change from manual count to computer count, gives us today the possibility to check hypotheses that have been made in the past about possible voting behaviours. In this section, we use the ballot data of the 1999, the 2001, the 2003, and the 2005 City Council and School Committee elections in Cambridge, Massachusetts, to estimate the number of voters who use a voting behaviour that has been predicted e.g. by Woodall (1983) and Tideman (2000). The predicted voting behaviour is a so-called *free riding* strategy. We describe free riding strategies in general and the free riding strategy predicted by Woodall and Tideman (section 3.1) and the one predicted by Hylland (1992) (section 3.2) in particular.

The “single transferable vote resists strategic voting” is the title of a well-known and frequently quoted paper by Bartholdi (1991). This title also reflects the opinion of many scientists. For example, Bowler (2000a) writes about tactical voting under STV:

> “To do so, voters would have to know the preference orderings of every other voter and the number of candidates being run. Even after they knew this information, voters then could not sit idly by but would have to begin to calculate the order in which candidates will get eliminated or elected and would also have to begin to conceive counterstrategies against all the other voters who would be similarly calculating what will happen if they altered their preference ordering over the parties. To put it mildly, this would seem an impossible task. In fact, STV generally presents such difficult calculations to voters seeking to behave tactically that it seems to make little sense to do anything other than register a sincere preference for the party that they would most like to see win.”

On the other side, there are many scientists who consider STV methods to be highly manipulable. This discrepancy is caused by the fact that the first group of scientists considers only strategies that are known from single-winner elections, while the second group of scientists considers *free riding* strategies, i.e. strategies that misuse the fact that in multi-winner elections, but not in single-winner elections, it is advantageous not to waste one’s vote by voting for a candidate who would be elected even without one’s vote.

Unfortunately, voters usually understand the usefulness of free riding strategies very fast so that e.g. it happens less and less frequently that candidates are elected with full quotas of first preferences. Brams (1996) and Kleinman (2003) observe a strong incentive of voters to rank more popular candidates insincerely low and less popular candidates insincerely high. Warren (1999a) observes that “one can get more out of one’s single vote by not giving one’s first preference to a handsomely supported candidate.” Hatton (1920) writes about the elections in 1919 to the 7 seats of the Kalamazoo City Council:

> “The first six candidates in number of first choice votes were elected and the seventh place went to Albert J. Todd who (with 3.4%) stood eleventh on the list. As a member of the commission the work of Todd had been efficient and thoroughly satisfactory. The small number of first choices which he received was apparently due to the expectation of a large majority of the voters that his re-election was a

> matter of course. As a result many who desired his re-election marked their ballots with a first-choice for candidates whom they favored, but whose success did not seem so well assured, and gave Todd a later choice. The result was a splendid vindication of the logic and accuracy of the Hare system.”

Weaver (1995) writes about the 17 elections 1927–1959 to the 7 seats of the Hamilton City Council:

> “As in most electoral systems, strategic voting appears to have affected outcomes. Over time surpluses of popular candidates seemed to shrink as their supporters learned to give their first-choice votes to candidates less assured of election.”

We call the two most important free riding strategies *Woodall free riding* and *Hylland free riding*, since Woodall (1983) and Hylland (1992) were the first ones who described these free riding strategies explicitly.

3.1. Woodall Free Riding

Woodall free riding is a useful strategy only for those STV methods where votes of eliminated candidates cannot be transferred to already elected candidates and therefore jump directly to the next highest ranked *hopeful* candidate (*leap-frogging*). A *Woodall free rider* is a voter who gives his first preference to a candidate who is believed by this voter to be eliminated early in the count even with this voter’s first preference. With this strategy, this voter assures that he does not waste his vote for a candidate who is elected already during the transfer of the initial surpluses.

Woodall (1983) writes:

> “The biggest anomaly is caused by the decision, always made, not to transfer votes to candidates who have already reached the quota of votes necessary for election. This means that the way in which a given voter’s vote will be assigned may depend on the order in which candidates are declared elected or eliminated during the counting, and it can lead to the following form of tactical voting by those who understand the system. If it is possible to identify a candidate *w* who is sure to be eliminated early (say, the Cambridge University Raving Loony Party candidate), then a voter can increase the effect of his genuine second choice by putting *w* first. For example, if two voters both want *a* as first choice and *b* as second, and *a* happens to be declared elected on the first count, then the voter who lists his choices as ‘$a \succ_v b \succ_v ...$’ will have (say) one third of his vote transferred to *b*, whereas the one who lists his choices as ‘$w \succ_v a \succ_v b \succ_v ...$’ will have all of his vote transferred to *b*, since *a* will already have been declared elected by the time *w* is eliminated. Since one aim of an electoral system should be to discourage tactical voting, this seems to me to be a serious drawback.”

Tideman (2000) writes:

> “People who understand STV well have developed a strategy for increasing the influence of their votes, based on this feature of the Newland-Britton (1997) rules. The strategy is to name as one’s first choice a candidate who is expected to be one of the first candidates

> excluded. All initial surpluses will have been transferred when this candidate is excluded, so none of the power of one’s vote is expended on electing candidates who can be elected without one’s help. Thus one can have greater influence over the remaining choices. Once, in the elections for the Board of the Electoral Reform Society, a candidate ran on the platform that he was the candidate for whom voters should vote if they did not want their votes wasted on someone who could be elected without their votes; he was nearly elected.”

However, Woodall free riding can be prevented by restarting the STV count with the remaining candidates whenever a candidate has been eliminated. Actually, the Meek (1969, 1970; Hill, 1987) method and the Warren (1994) method do this. Therefore, Woodall (1983) and Tideman (1995, 2000) suggest that one of these methods should be used.

A good test for Woodall free riding is an STV election with *write-in options* (i.e. with the possibility for the voters to vote for any person by writing this person’s name on the ballot). The City Council and the School Committee of Cambridge, Massachusetts, are elected by a traditional STV method that is vulnerable to Woodall free riding and that has write-in options. In the elections to the 9 seats of the City Council, the voter can vote for up to 9 write-ins. In the elections to the 6 seats of the School Committee, the voter can vote for up to 6 write-ins. Here the optimal Woodall free riding strategy is to give one’s first preference to a completely unimportant write-in.

In table 3.1.1, row “1” contains the numbers of voters in the 1999 City Council elections (column “CC 1999”), in the 1999 School Committee elections (“SC 1999”), in the 2001 City Council elections (“CC 2001”), in the 2001 School Committee elections (“SC 2001”), in the 2003 City Council elections (“CC 2003”), in the 2003 School Committee elections (“SC 2003”), in the 2005 City Council elections (“CC 2005”), and in the 2005 School Committee elections (“SC 2005”) in Cambridge, Massachusetts. Row “2” contains the numbers of voters who cast a valid first preference for a write-in. Row “3” contains the numbers of voters who have to be subtracted from row “2” because they cast preferences only for write-ins and who are therefore obviously not Woodall free riders. Furthermore, those voters who do not cast at least a valid second and a valid third preference have to be subtracted (row “4”) because these voters cannot be Woodall free riders. Therefore, row “5” contains the numbers of voters who could be write-in Woodall free riders.

	CC 1999	SC 1999	CC 2001	SC 2001	CC 2003	SC 2003	CC 2005	SC 2005
1	18,613	17,796	17,125	16,488	20,080	18,696	16,068	15,468
2	28	26	30	51	38	98	15	55
3	9	5	12	32	16	66	9	36
4	0	4	0	2	3	8	0	9
5	19	17	18	17	19	24	6	10

Table 3.1.1: Potential write-in Woodall free riders in the 1999, the 2001, the 2003, and the 2005 elections to the City Council and the School Committee of Cambridge, Massachusetts

In all six elections, the numbers of voters who could be write-in Woodall free riders are only about 0.1%. In table 3.1.2, N is the number of voters, $T_1(b)$ is the number of voters who cast a valid first preference for candidate b, $T_2(b)$ is the number of voters who cast a valid first preference for candidate *b* and at least also a valid second preference, *U* is the number of potential write-in Woodall free riders, and $U(b)$ is the number of potential write-in Woodall free riders who cast a valid second preference for candidate *b*. Table 3.1.2. lists $T_1(b)$, $T_2(b)$, and $U(b)$ for all those candidates *b* who are elected before candidates have to be eliminated. Only in 7 of these 18 cases, $U(b)/U$ is larger than $T_2(b)/N$. Therefore, also the voters in column “*U*” seem to be no Woodall free riders because otherwise super-proportionally many of these voters would have cast a second preference for a candidate who reached the quota before candidates had to be eliminated.

	election	candidate *b*	*N*	$T_1(b)$	$T_1(b)/N$	$T_2(b)$	$T_2(b)/N$	*U*	$U(b)$	$U(b)/U$
1	CC 1999	AD Galluccio	18,613	2,705	14.5%	2,515	13.5%	19	2	10.5%
2	SC 1999	AL Turkel	17,796	2,617	14.7%	2,360	13.3%	17	2	11.8%
3	CC 2001	H Davis	17,125	1,713	10.0%	1,645	9.6%	18	2	11.1%
4	CC 2001	B Murphy	17,125	1,716	10.0%	1,627	9.5%	18	1	5.6%
5	CC 2001	AD Galluccio	17,125	3,230	18.9%	2,947	17.2%	18	5	27.8%
6	SC 2001	JG Grassi	16,488	2,135	12.9%	1,728	10.5%	17	0	0.0%
7	SC 2001	AB Fantini	16,488	2,854	17.3%	2,353	14.3%	17	1	5.9%
8	SC 2001	AL Turkel	16,488	2,862	17.4%	2,484	15.1%	17	4	23.5%
9	CC 2003	AD Galluccio	20,080	2,994	14.9%	2,757	13.7%	19	1	5.3%
10	SC 2003	JG Grassi	18,696	2,295	12.3%	1,794	9.6%	24	0	0.0%
11	SC 2003	R Harding	18,696	2,362	12.6%	1,806	9.7%	24	5	20.8%
12	SC 2003	B Lummis	18,696	2,604	13.9%	2,252	12.0%	24	6	25.0%
13	SC 2003	MC McGovern	18,696	2,716	14.5%	2,303	12.3%	24	2	8.3%
14	SC 2003	AB Fantini	18,696	2,905	15.5%	2,385	12.8%	24	1	4.2%
15	SC 2003	N Walser	18,696	3,842	20.5%	3,077	16.5%	24	7	29.2%
16	CC 2005	AD Galluccio	16,068	2,001	12.5%	1,828	11.4%	6	0	0.0%
17	SC 2005	AB Fantini	15,468	2,281	14.7%	1,858	12.0%	10	1	10.0%
18	SC 2005	PM Nolan	15,468	2,387	15.4%	2,139	13.8%	10	4	40.0%

Table 3.1.2: $T_1(b)$, $T_2(b)$, and $U(b)$ for each candidate *b* who is elected before candidates have to be eliminated

$T(a,b)$ is the number of voters who cast a valid first preference for candidate *a*, a valid second preference for candidate *b*, and at least also a valid third preference. Woodall free riding is a useful strategy only when one has at least a sincere first and a sincere second preference. A given voter can be a Woodall free rider only when he casts at least a valid first, a valid second, and a valid third preference. When a given voter, whose sincere first preference is candidate *b*, uses Woodall free riding, then $T_2(b)$ decreases and for some other candidate *a*, who is eliminated early in the count, $T(a,b)$ increases. Therefore, another good test for Woodall free riding is to calculate $T(a,b)$ for each pair of candidates. If (1) $T(a,b)/T_1(a)$ is large compared to $T_2(b)/N$ and (2) $T(a,b)/T_1(a)$ decreases with increasing $T_1(a)$ for those pairs of candidates where candidate *a* is eliminated early in the count and candidate *b* is elected before candidates have to be eliminated, then this is an evidence that voters use Woodall free riding.

Tables 3.1.3 – 3.1.10 contain $T(a,b)$ for each pair of candidates *a* and *b* where candidate *b* is elected before candidates have to be eliminated. “$T_1(a)$” contains the numbers of voters who cast a valid first preference for the candidate in column “candidate *a*”. The column “Galluccio” (resp. “Turkel”, resp. “Davis”, etc.) contains the numbers of voters of column “$T_1(a)$” who cast a valid second preference for Galluccio (resp. Turkel, resp. Davis, etc.) and at least also a valid third preference.

In tables 3.1.3 – 3.1.10, $T(a,b)/T_1(a)$ rather increases than decreases with increasing $T_1(a)$. Also the prediction, that $T(a,b)/T_1(a)$ is large compared to $T_2(b)/N$, is not fulfilled. This is surprising because, in so far as Woodall free riding certainly is a useful strategy, one would expect that at least some voters use this strategy. A possible explanation, why voters do not use Woodall free riding, is that they fear that, when too many voters give their first preference to candidate *a* because they believe that he is eliminated early in the count, then it could happen that candidate *a* gets so many votes that he is elected (Fennell, 1994; Hill, 1994; Tideman, 2000). But this can only explain why $T(a,b)/T_1(a)$ does not decrease so fast with increasing $T_1(a)$; this cannot explain why $T(a,b)/T_1(a)$ increases with increasing $T_1(a)$. A possible explanation, why $T(a,b)/T_1(a)$ increases with increasing $T_1(a)$, is that voters are confronted with two problems:

1. It is a useful strategy not to waste one’s vote by voting for a candidate *b* who is elected even without one’s vote. However, when too many voters use Woodall free riding and cast a first preference for candidate *a*, because they believe that he is eliminated early in the count even with one’s vote, then it could happen that candidate *a* gets so many votes that he is elected.

2. It is a useful strategy not to vote for a candidate *a* who is believed to be eliminated with a high probability even with one’s vote, because otherwise there is the danger that there are no acceptable candidates anymore to whom this voter could transfer his vote when candidate *a* is eliminated. When a voter votes for a candidate *a*, who is eliminated with a high probability even with this voter’s vote, and not for candidate *b*, who is less preferred but who has great chances of being elected when he is not eliminated early, then there is the great danger that candidate *b* is eliminated before candidate *a* is eliminated, so that this voter cannot transfer his vote to candidate *b* anymore when candidate *a* is eliminated.

Because of problem 2, only those voters, who cannot identify themselves with any of the stronger candidates, vote for candidates who are believed to be eliminated with a high probability; therefore, $T(a,b)/T_1(a)$ is low for low $T_1(a)$ for those candidates *b* who are elected before candidates have to be eliminated; therefore, $T(a,b)/T_1(a)$ rather increases than decreases with increasing $T_1(a)$.

candidate a	$T_1(a)$	Galluccio
CO Christenson	28	2 (7.1%)
DP Wormwood-Malone	28	0 (0.0%)
WC Jones	31	2 (6.5%)
AK Nidle	40	0 (0.0%)
VL Dixon	44	3 (6.8%)
JJ Chase	102	10 (9.8%)
DM Giacobbe	109	22 (20.2%)
JM Williamson	128	2 (1.6%)
R Winters	301	27 (9.0%)
H Peixoto	308	46 (14.9%)
D Hoicka	325	7 (2.2%)
EC Snowberg	425	12 (2.8%)
D Trumbull	533	129 (24.2%)
R Goodwin	805	296 (36.8%)
DP Maher	1,030	309 (30.0%)
K Triantafillou	1,167	42 (3.6%)
MA Sullivan	1,321	278 (21.0%)
KE Reeves	1,420	149 (10.5%)
H Davis	1,458	70 (4.8%)
J Braude	1,480	50 (3.4%)
TJ Toomey	1,497	233 (15.6%)
MC Decker	1,642	43 (2.6%)
KL Born	1,658	100 (6.0%)

Table 3.1.3: Potential Woodall free riders in the 1999 City Council elections in Cambridge, Massachusetts

candidate a	$T_1(a)$	Turkel
SM Burke	212	6 (2.8%)
JF Patterson	278	9 (3.2%)
AE Thompson	373	35 (9.4%)
ML Brazo	471	82 (17.4%)
D Harding	698	24 (3.4%)
ET Kenney	738	134 (18.2%)
M Harshbarger	1,550	109 (7.0%)
N Walser	1,894	520 (27.5%)
SM Segat	1,985	480 (24.2%)
JG Grassi	2,269	97 (4.3%)
AB Fantini	2,277	55 (2.4%)
D Simmons	2,408	506 (21.0%)

Table 3.1.4: Potential Woodall free riders in the 1999 School Committee elections in Cambridge, Massachusetts

candidate *a*	$T_1(a)$	Davis	Murphy	Galluccio	sum
JM Williamson	58	2 (3.4%)	2 (3.4%)	3 (5.2%)	7 (12.1%)
JE Condit	63	6 (9.5%)	0 (0.0%)	5 (7.9%)	11 (17.5%)
H Peixoto	69	5 (7.2%)	3 (4.3%)	7 (10.1%)	15 (21.7%)
VL Dixon	92	2 (2.2%)	3 (3.3%)	7 (7.6%)	12 (13.0%)
RL Hall	153	3 (2.0%)	13 (8.5%)	18 (11.8%)	34 (22.2%)
J Horowitz	155	14 (9.0%)	12 (7.7%)	6 (3.9%)	32 (20.6%)
SE Jens	278	8 (2.9%)	5 (1.8%)	35 (12.6%)	48 (17.3%)
S Iskovitz	345	29 (8.4%)	30 (8.7%)	9 (2.6%)	68 (19.7%)
EA King	378	43 (11.4%)	46 (12.2%)	25 (6.6%)	114 (30.2%)
DP Maher	1,017	32 (3.1%)	41 (4.0%)	304 (29.9%)	377 (37.1%)
J Pitkin	1,091	222 (20.3%)	202 (18.5%)	48 (4.4%)	472 (43.3%)
KE Reeves	1,141	72 (6.3%)	34 (3.0%)	125 (11.0%)	231 (20.2%)
MA Sullivan	1,315	45 (3.4%)	28 (2.1%)	316 (24.0%)	389 (29.6%)
D Simmons	1,339	186 (13.9%)	137 (10.2%)	74 (5.5%)	397 (29.6%)
TJ Toomey	1,402	44 (3.1%)	11 (0.8%)	272 (19.4%)	327 (23.3%)
MC Decker	1,540	298 (19.4%)	215 (14.0%)	163 (10.6%)	676 (43.9%)
H Davis	1,713	---	254 (14.8%)	114 (6.7%)	
B Murphy	1,716	343 (20.0%)	---	105 (6.1%)	
AD Galluccio	3,230	137 (4.2%)	90 (2.8%)	---	

Table 3.1.5: Potential Woodall free riders in the 2001 City Council elections in Cambridge, Massachusetts

candidate *a*	$T_1(a)$	Grassi	Fantini	Turkel	sum
VJ Delaney	240	23 (9.6%)	29 (12.1%)	5 (2.1%)	57 (23.8%)
F Baker	324	28 (8.6%)	62 (19.1%)	9 (2.8%)	99 (30.6%)
ML Erlien	1,193	21 (1.8%)	25 (2.1%)	272 (22.8%)	318 (26.7%)
SM Segat	1,590	61 (3.8%)	107 (6.7%)	619 (38.9%)	787 (49.5%)
N Walser	1,677	42 (2.5%)	68 (4.1%)	596 (35.5%)	706 (42.1%)
R Harding	1,689	172 (10.2%)	156 (9.2%)	176 (10.4%)	504 (29.8%)
AC Price	1,873	41 (2.2%)	71 (3.8%)	319 (17.0%)	431 (23.0%)
JG Grassi	2,135	---	698 (32.7%)	94 (4.4%)	
AB Fantini	2,854	942 (33.0%)	---	158 (5.5%)	
AL Turkel	2,862	97 (3.4%)	133 (4.6%)	---	

Table 3.1.6: Potential Woodall free riders in the 2001 School Committee elections in Cambridge, Massachusetts

candidate *a*	$T_1(a)$	Galluccio
DJ Greenwood	39	1 (2.6%)
VL Dixon	64	9 (14.1%)
RL Hall	96	11 (11.5%)
RJ LaTrémouille	126	3 (2.4%)
L Taymorberry	188	2 (1.1%)
EA King	361	46 (12.7%)
AL Smith	480	5 (1.0%)
CK Bellew	735	61 (8.3%)
CA Kelley	992	98 (9.9%)
J Pitkin	1,010	49 (4.9%)
D Simmons	1,181	38 (3.2%)
DP Maher	1,190	340 (28.6%)
MS DeBergalis	1,206	54 (4.5%)
B Murphy	1,362	72 (5.3%)
MC Decker	1,378	146 (10.6%)
KE Reeves	1,525	127 (8.3%)
TJ Toomey	1,613	330 (20.5%)
MA Sullivan	1,656	404 (24.4%)
H Davis	1,846	136 (7.4%)

Table 3.1.7: Potential Woodall free riders in the 2003 City Council elections in Cambridge, Massachusetts

candidate *a*	$T_1(a)$	Grassi	Harding	Lummis	McGovern	Fantini	Walser	sum
C Craig	573	25 (4.4%)	20 (3.5%)	169 (29.5%)	89 (15.5%)	59 (10.3%)	47 (8.2%)	409 (71.4%)
AC Price	1,301	27 (2.1%)	180 (13.8%)	241 (18.5%)	147 (11.3%)	64 (4.9%)	360 (27.7%)	1,019 (78.3%)
JG Grassi	2,295	---	229 (10.0%)	43 (1.9%)	77 (3.4%)	651 (28.4%)	146 (6.4%)	
R Harding	2,362	285 (12.1%)	---	182 (7.7%)	161 (6.8%)	192 (8.1%)	244 (10.3%)	
B Lummis	2,604	71 (2.7%)	217 (8.3%)	---	637 (24.5%)	96 (3.7%)	356 (13.7%)	
MC McGovern	2,716	111 (4.1%)	238 (8.8%)	614 (22.6%)	---	191 (7.0%)	355 (13.1%)	
AB Fantini	2,905	933 (32.1%)	176 (6.1%)	102 (3.5%)	155 (5.3%)	---	165 (5.7%)	
N Walser	3,842	179 (4.7%)	509 (13.2%)	468 (12.2%)	318 (8.3%)	253 (6.6%)	---	

Table 3.1.8: Potential Woodall free riders in the 2003 School Committee elections in Cambridge, Massachusetts

candidate *a*	$T_1(a)$	Galluccio
JE Condit	42	1 (2.4%)
RL Hall	75	6 (8.0%)
RJ LaTrémouille	118	2 (1.7%)
AL Green	181	10 (5.5%)
B Hees	198	12 (6.1%)
LJ Adkins	243	11 (4.5%)
JA Gordon	626	31 (5.0%)
DP Maher	902	204 (22.6%)
S Seidel	973	46 (4.7%)
CA Kelley	1,042	72 (6.9%)
KE Reeves	1,207	110 (9.1%)
B Murphy	1,236	50 (4.0%)
D Simmons	1,330	62 (4.7%)
TJ Toomey	1,432	260 (18.2%)
H Davis	1,459	68 (4.7%)
MA Sullivan	1,464	318 (21.7%)
MC Decker	1,524	163 (10.7%)

Table 3.1.9: Potential Woodall free riders in the 2005 City Council elections in Cambridge, Massachusetts

candidate *a*	$T_1(a)$	Fantini	Nolan	sum
MC McGovern	1,413	82 (5.8%)	230 (16.3%)	312 (22.1%)
B Lummis	1,514	77 (5.1%)	170 (11.2%)	247 (16.3%)
L Schuster	1,843	63 (3.4%)	362 (19.6%)	425 (23.1%)
R Harding	1,981	165 (8.3%)	115 (5.8%)	280 (14.1%)
JG Grassi	1,990	559 (28.1%)	58 (2.9%)	617 (31.0%)
N Walser	2,004	191 (9.5%)	324 (16.2%)	515 (25.7%)
AB Fantini	2,281	---	87 (3.8%)	
PM Nolan	2,387	86 (3.6%)	---	

Table 3.1.10: Potential Woodall free riders in the 2005 School Committee elections in Cambridge, Massachusetts

3.2. Hylland Free Riding

Problem 1 can be circumvented by using Hylland free riding instead of Woodall free riding. Unlike Woodall free riding, Hylland free riding can be used under any STV method. Hylland writes (1992):

> “Both for groups and for individual voters it could be advantageous not to vote for a candidate who is considered certain of winning election, even if that candidate is one’s first choice. Suppose that my true first and second choices are *a* and *b*, I am sure *a* will get many more first preferences than needed for election, but I find *b*’s chances uncertain. If I list *a* as the first preference on my ballot, its weight is reduced before it reaches *b*. If I omit *a*, *b* gets a vote with full weight.”

In short, a Hylland free rider is a voter who omits in his individual ranking completely all those candidates who are certain to be elected. Of course, when too many voters use Hylland free riding then it can happen that the candidate with the cast first preference is elected while the candidate with the sincere first preference is eliminated. However, when a voter uses Hylland free riding then the candidate with the cast first preference is one of this voter’s favorite candidates, while when this voter uses Woodall free riding then the candidate with the cast first preference is a candidate who this voter does not want to be elected. Therefore, although both free riding strategies can backfire, such a backfire is less severe under Hylland free riding than under Woodall free riding.

Problem 2 can be circumvented by voting only for those candidates who are believed to be in the race until the final count. In so far as a candidate will be in the final count when he has more than $N/(M+2)$ first preferences, it is a useful strategy to cast one’s first preference only for one of those candidates who are believed to get between $N/(M+2)$ and $N/(M+1)$ first preferences.

This voting behaviour could best be observed in Canada, because here the city councils were elected by a traditional STV method for a one year term and in a single city-wide district so that the voters had very precise information about the support of the different candidates. A consequence of this voting behaviour was that usually almost all first preferences were concentrated on $M+1$ almost equally strong candidates (Berger, 2004; Harris, 1930; Johnston, 2000; Pilon, 1994), so that the weakest of these $M+1$ candidates was eliminated and the winners happened to be those M candidates with the largest numbers of first preferences. Johnston (2000) writes that one of the main criticisms of STV was that it was “one of the most common features of PR in Canadian municipal elections” that “the final count closely mirrored the results of the first count”. Pilon (1994) writes that the main problem of STV in Canada was that it “did not seem to make much difference in the results. After days of counting, eliminating candidates, and transferring fractions of support from one aspirant to another, there was little difference between the first choice results and the final tally.” And Berger (2004) writes: “Complexity and the discovery that STV elected in most cases the same persons as would have been elected had only first preferences been counted, were the two most frequently given explanations for public indifference and support for repeal.”

The same voting behaviour was also observed in the Isle of Man. The lower house (*House of Keys*) was elected by a traditional STV method. Also here the winners always happened to be those candidates with the largest numbers of first preferences. This led to the abolition of STV in 1995 (Herbert, 2003).

3.3. Summary

A *free rider* is a voter who misuses the fact that, in multi-winner elections, it is a useful strategy not to vote for a candidate who will be elected even without one’s vote. Free riding is a very serious problem of STV methods. The two free riding strategies that have been predicted in the literature are Woodall (1983) free riding and Hylland (1992) free riding.

A *Woodall free rider* is a voter who gives his first preference to a candidate who is believed by this voter to be eliminated early in the count even with this voter’s first preference; with this strategy, this voter assures that he does not waste his vote for a candidate who is elected already during the transfer of the initial surpluses. A *Hylland free rider* is a voter who omits in his individual ranking completely all those candidates who are certain to be elected.

It is not possible to extract the number of Hylland free riders simply from the ballot data. But with additional assumptions, it is possible to extract the number of Woodall free riders. We used the ballot data of the 1999, the 2001, the 2003, and the 2005 City Council and School Committee elections in Cambridge, Massachusetts, to estimate the number of voters who use Woodall free riding. We could not find any evidence at all that voters use this strategy. Possible explanations, why voters do not use this strategy, are:

(a) When too many voters cast a first preference for candidate *a*, not because he is their sincere first preference but because they believe that he will be eliminated early in the count, it could happen that this candidate gets so many votes that he is elected (Fennell, 1994; Hill, 1994; Tideman, 2000).

(b) It is not useful to vote for a candidate *a* who is eliminated with a high probability, because it could happen that there are no acceptable candidates anymore to whom this voter could transfer his vote when candidate *a* is eliminated.

(c) When a voter considers his second favorite candidate to be only slightly worse than his favorite candidate, then Hylland (1992) free riding is less dangerous than Woodall (1983) free riding, in so far as a backfire is less severe under Hylland free riding than under Woodall free riding.

(d) The political organizations have not yet found a simple way to use Woodall free riding on a larger scale to increase their numbers of seats. Therefore, the voters are usually not pointed to this strategic problem.

On the other side, we quoted several empirical papers on STV that demonstrate that Hylland free riding is a frequently used strategy (Berger, 2004; Harris, 1930; Johnston, 2000; Pilon, 1994).

4. Vote Management

> *Vote management is a practice that seems to have acquired the same degree of mystique as alchemy, a technique that, in some eyes, can be used to conjure a seat out of even the most meagre bundle of first preferences.*
>
> Michael Gallagher (1993a)

In multi-winner elections, the term *vote management* (or *vote allocation*) refers to strategies where a party divides the electorate into parts and asks all the voters of the same part to vote in the same manner (prescribed by this party in advance and for each part in a different manner). Usually, these parts are also known as *pockets* or *bailiwicks* and this strategy is also known as *bailiwicking* only when the electorate is divided according to geographic or social criteria; but for the sake of simplicity, we will use these terms also when the electorate is divided according to other criteria. Under STV methods, this strategy is also known as *spread the preferences* (STP) (Farrell, 1993, 2000).

Vote management is also used e.g. under the single non-transferable vote (SNTV) (Cox, 1997; Grofman, 1999; Lijphart, 1986), under limited voting (Hoag, 1926; Lijphart, 1986), and under cumulative voting (Bowler, 2003). However, the aim of this paper is to give an overview over vote management strategies mainly under STV methods (Barber, 1995; Bax, 1973; Bowler, 1991a, 1991b, 2000b; Busteed, 1990; Farrell, 1992, 1996, 2000; Gallagher, 1990b, 1992, 1993b, 2003b; Laver, 1987, 1998; Mair, 1987a, 1987b; Parker, 1982; Penniman, 1987; Sacks, 1970, 1976).

In section 4.1, typical examples for vote management strategies are presented. In sections 4.2 – 4.6, those 5 questions, that have to be answered by that party that wants to run a vote management strategy successfully, are discussed.

4.1. Some Examples

A typical example looks as follows (Busteed, 1990; Mair, 1987a, 1987b) (*Taoiseach* = Prime Minister; *Tánaiste* = Deputy Prime Minister; *Seanad* = upper house; *Dáil* = lower house; TD = *Teachta Dála* = member of the Dáil):

> "1921–1982 the Corish family had held a seat for Labour in the Wexford constituency, but on the retirement of the Labour TD Brendan Corish (Labour leader 1960–1977 and Tánaiste 1973–1977) in February 1982 his brother Desmond Corish had failed to hold the seat, and it had gone to Fianna Fáil so that this 5-seat constituency (65,000 registered voters) had resulted in three seats for Fianna Fáil and two seats for Fine Gael. Suspecting that one of the Fianna Fáil seats was vulnerable, Fine Gael proceeded on two fronts. It first decided to breach a long-standing local agreement that it would never seriously challenge a Labour candidate based on the Corish family home base of Wexford Town. Because both Fine Gael TDs came from the north and centre of this constituency, the Constituency Review Committee recommended that a third candidate be nominated from the southern end, the area that included the towns of Wexford (where 18,000 voters were registered) and New Ross (where 8,000 voters were registered). With a reasonably strong potential candidate from this area, Avril Doyle, the first lady mayor of Wexford Town and

> from a family long involved in Fine Gael, the party believed that her nomination and a more judicious division of first preferences between the two incumbents could give the party a third seat. The strategy worked perfectly. Although Fine Gael's overall vote share rose, the number of first preferences won by its two incumbent TDs Michael Joseph d'Arcy and Ivan Yates fell slightly, as intended (Fine Gael: Feb. 1982 37.7% [3.3% to the unsuccessful candidate Louise Hennessy], Nov. 1982 41.4%; d'Arcy: Feb. 1982 18.0%, Nov. 1982 15.8%; Yates: Feb. 1982 16.4%, Nov. 1982 14.2%). Both had sufficient votes to ensure their own election, but the reduction in their total gave Doyle (11.4%) sufficient first preferences to remain ahead of the Labour candidate (Brendan Howlin: 9.9%). Because Labour's transfers were likely to go predominantly to Fine Gael (as would Fine Gael's go to Labour in similar circumstances), Doyle's consistent plurality over the Labour candidate meant that she would benefit from his earlier elimination. This followed the fourth count, when Doyle had accumulated 6,140 (12.2%) against 5,481 (10.9%) votes for the Labour candidate. After his elimination, 4,364 lower-preference votes passed to Fine Gael, a transfer sufficient to push the two incumbents past the quota while at the same time placing Doyle close enough to be assured of election."

Gallagher (1999) describes the vote management strategies during the Dáil elections in 1997:

> "Both major parties sought to manage the vote in specific constituencies. For example, in the five-seater Dun Laoghaire, a Fianna Fáil target seat, the long-serving TD David Andrews had often taken too high a percentage of the Fianna Fáil vote (72.5% in 1992), and his running mate had trailed too far behind to be elected. Consequently, with Andrews' agreement, the local organisation divided the constituency into two, and voters were sent or handed items of campaign literature asking them to vote in particular ways. One, headed 'Fianna Fáil Vote Management Strategy', showed a map of the constituency divided into two areas so that all voters would know which candidate to give their first preference to in order to maximise the party's chances of winning two seats. In the event, the balance between the two candidates was more even than at previous elections, and Fianna Fáil gained a second seat.
>
> Meanwhile, Galway East had changed from a three-seater to a four-seater, and Fianna Fáil, Fine Gael, and the Progressive Democrats all had a good chance of taking the additional seat. Fine Gael devised what was termed the 'railway line strategy', based on the Dublin-Galway line that ran from east to west across the middle of the constituency. Voters living north of the line were asked to give their first preference to Paul Connaughton and their second to Ulick Burke; those living to the south were asked to switch those preferences. This plan required some sacrifice on the part of Connaughton, the incumbent TD, who had built up a degree of personal support south of the line by his constituency work and who in a free and open contest would have been certain of re-election. The strategy worked well and was crucial in Fine Gael's capture of the additional seat."

Gallagher (2003a) describes the vote management strategies during the Dáil elections in 2002:

> “There were a number of examples of vote management schemes in 2002. Sometimes parties explicitly called on their voters to vote a certain way. For example, in Wexford, where Fianna Fáil was making a determined effort to take three seats, the party produced a newspaper advertisement showing a map of the constituency, with different areas shaded in different colours. Those living in the areas coloured yellow, basically the central belt, were asked to give a first preference to John Browne; those living in a purple area (the north-east and the south-east) were asked to vote for Tony Dempsey, and those in a green-coloured area (the south-west and north-west) were asked to vote first for Hugh Byrne. In the event the vote was reasonably well balanced but the effort was in vain as the party simply didn’t receive enough votes, and it was the independent Liam Twomey who took the seat that Fine Gael lost.
>
> In the same constituency, vote management within the Fine Gael camp ran less smoothly. Avril Doyle, a former TD and now an MEP, had been persuaded to return to fight the election sine Fine Gael was in great danger of losing a seat given that its long-standing incumbent Ivan Yates was retiring. However, the other candidates were not keen to allow her as much territory as she wanted. She appealed to the party’s national headquarters, and it duly barred Paul Kehoe (Yates’ successor) from canvassing in an area north-east of Wexford town. Doyle explained, ‘I insisted that I have the Wexford district’, pointing out that she lived there. Kehoe, who had previously been canvassing there, was not pleased, but said he would abide by the decision, and there was some surprise and resentment that Doyle had appealed to the national body rather than having the dispute resolved within the constituency organisation. In the event Kehoe was elected and Doyle finished last of the three Fine Gael candidates.
>
> The five-seat Mayo constituency saw plenty of turf wars — sometimes reminiscent of great power disputes during the nineteenth century — within both major parties. Fights broke out between supporters of rival Fianna Fáil candidates Beverley Cooper-Flynn and Tom Moffatt over who had the right to hold a collection outside a church gate in Foxford. Cooper-Flynn explained that the four Fianna Fáil candidates had signed up to an agreement about who had the right to which territory, but within two hours ‘people were breaking it all over the place’; she herself was still respecting it, she said, ‘for the moment anyway’. Things were no better within Fine Gael. The vote management scheme there entailed damping down the vote of the frontrunner, Michael Ring, and boosting support for the two other incumbents, Jim Higgins and Enda Kenny, who were seen as more vulnerable. The Fine Gael constituency organisation thus decided that Ring would be ‘kept out of’ certain areas in south Mayo, even though he had been holding monthly clinics there over the previous five years. Ring was said to be further displeased when he was asked to relinquish some ground on the east side of Erris to the party’s fourth candidate Ernie Caffrey, in exchange for some presumably less promising territory around Foxford.”

Gallagher (2008a) describes the vote management strategies during the Dáil elections in 2007:

> "Characteristically, vote management entails dividing the constituency geographically and, like handing out slices of a pie, awarding sections of it to individual candidates. In urban areas, an alternative approach that keeps candidates apart without formally dividing the territory involves allowing each candidate to canvass the whole constituency but making sure that only one candidate is in a particular area at a time. Candidates may be told which areas they are and are not permitted to canvass, and voters may be asked to vote in a particular way depending on which part of the constituency they live in. For example, in the Wexford constituency Fianna Fáil awarded each of its three candidates sole canvassing rights in their home area, while the Gorey area was divided among them down to the level of individual polling boxes. Within Fine Gael, the New Ross electoral area was open territory, and, moreover, each candidate was allowed to canvass one day a week in the other candidates' areas provided this was arranged in advance. Candidates will, of course, expect to be given the area around their home base, and so the borders of their respective bailiwicks, like a disputed frontier between states, are where trouble is most likely to flare. Matters can become particularly tense if one candidate is perceived as a front-runner, in which case vote management requires trying to siphon some of his or her support to the weaker member of the team. As well as provoking resistance from the front-runner, this tactic, if pushed too far, might have the unintended effect of leading to the election of the weaker candidate at the expense of the front-runner, thus costing the party one of its leading lights, as happened in Cork North-West in 2002 when a Fine Gael vote management plan backfired and the front-bencher Michael Creed lost his seat as a result.
>
> When party support seems to be slipping candidates become edgy and the normal courtesies of intra-party conduct may be cast to the winds. In Cork East the two Fianna Fáil incumbents took their gloves off in the run-up to polling day. Michael Ahern, based in the south of the constituency, appealed for support from the northern area, the bailiwick of Ned O'Keeffe. Traditional boundaries, he said, meant nothing in this election: 'At the last election there was a boundary in place which he [O'Keeffe] broke lock, stock and barrel right up to my doorstep. This time there is no boundary so it is open territory.' O'Keeffe, who had had to resign a junior ministerial position in 2001 and had then suffered the further pain of seeing Ahern appointed to the equivalent position in June 2002, said that if Ahern had performed competently in his ministerial post Fianna Fáil would have been pressing for three seats in the constituency instead of having to worry about holding on to two. Ahern rounded off the exchange by opining that some of O'Keeffe's comments amounted to slander, though he did not intend to take legal action. In the event, as at every election since November 1982, both Ahern and O'Keeffe were comfortably re-elected.
>
> In Dublin Central vote management takes on a different meaning altogether. Fianna Fáil usually wins around 40% of the votes here, making it competitive for two seats. Whereas the logic outlined above would suggest something like an even division of the votes between two candidates, the local constituency organisation, dominated by party leader Bertie Ahern, operates instead the tactic of trying to

> maximise Ahern's first preference support in the hope that enough votes will transfer from his surplus on to his running mate(s) to secure a second seat. Neutral observers believe that when Fianna Fáil has taken two seats in Dublin Central it has been despite rather than because of this singular approach, but, undaunted, the organisation repeated the tactic in 2007. Ahern received 12,734 first preferences, 83% of the party total. On this occasion he had two running mates — reportedly this was simply because he had been unable to decide which one to drop from the ticket, and so let them both stand — and they were in effect competing for Ahern's second preferences, with the winner likely to take a seat. Although Mary Fitzpatrick received more first preferences than Cyprian Brady (1,725 compared with 939), Brady, a long-time member of Ahern's constituency organisation, received over 1,000 more of Ahern's second preferences than she did and went on to take the seat. It turned out that the party organization had distributed 30,000 leaflets early in the morning of election day asking voters to vote 1-2-3 in that order for Ahern, Brady and Fitzpatrick. While, Fitzpatrick said, 'I never thought they were the Legion of Mary', she had not expected the party to 'shaft' and 'undermine' her as it had. However, one of Ahern's associates in the constituency, Chris Wall, explained that Fitzpatrick had brought her fate upon herself by firing the first shot: she had distributed campaign literature in some areas asking voters to give her their first preference. Wall said: 'She was asked not to do this sort of thing. Having then done it, she therefore effectively set in train a motion she wasn't going to be able to stop.'"

Vote management is neither an invention of the 1970s or 1980s nor a pure Irish phenomenon. This strategy can be observed everywhere where STV is being used. In the history of vote management under STV, that example that attracted most attention was the vote management strategy of the Republican Party during the elections to the Cincinnati City Council in 1925. The Cincinnati City Council was elected by STV in a single 9-seat constituency. The Republican Party hoped to win 6 of the 9 seats. Therefore, it divided Cincinnati into 6 bailiwicks and nominated 6 candidates *a*, *b*, *c*, *d*, *e*, and *f*. It asked its supporters to vote $a \succ_v b \succ_v ...$ in the first bailiwick, $b \succ_v a \succ_v ...$ in the second bailiwick, $c \succ_v d \succ_v ...$ in the third bailiwick, $d \succ_v c \succ_v ...$ in the fourth bailiwick, $e \succ_v f \succ_v ...$ in the fifth bailiwick, and $f \succ_v e \succ_v ...$ in the sixth bailiwick. However, the strategy of the Republican Party failed mainly because it overestimated its support. Goldman (1930) writes:

> "The regular Republican organization has been experimenting, but has apparently not yet found a way to 'beat the system.' In the first election it thought that it would gain strength by concentrating on only six candidates. But it found that this was a source of weakness, because it had fewer candidates than it might have had to rally its friends to the tickets and so pass on additional votes to its leading candidates when they were eliminated. It also attempted to district the city by wards, by dictating to its voters in each ward, not only for what group of candidates they should vote, but in what order they should vote them. In this it failed signally in several wards, because the voters would not accept the dictated first-choice candidate. It also tried to pair candidates, with the result that only one of each pair was elected, and not the strongest candidate."

Harris (1930) writes:

> “The Republican organization was obviously puzzled to know what tactics to adopt to win the election. Not understanding the fundamentals of PR, it decided to put only six candidates in the field, and to assign each to a particular district. It was believed that this arrangement would permit the organization to beat the system. The candidates were also assigned in pairs, each candidate being instructed to solicit second-choice votes for the candidate with whom he was paired. As the campaign progressed, this led to a great deal of friction. Dissension broke out within the ranks, for certain candidates thought they were being double crossed, and others believed that they were assigned to difficult districts. Then, too, many voters resented being told in what order to vote, and this lost votes. The assignment was unhappy in several instances. For example, Adolph Kummer, President of the Central Labor Union, was assigned to a district which contained some of the wealthiest residential sections of the city. The favored organization candidates were assigned to the Republican strongholds.”

Kolesar (1995) writes:

> “The Republican organization waged an inept campaign. Rather than nominate a full slate, they chose only six candidates. They ‘districted’ the city among the six, but, ‘as the fight got hotter and the Republican nominees saw that the machine could not assure the victory of all six candidates, they commenced to fight among themselves. Friends of Republican candidates sought first choice votes in districts that had been apportioned to other Republican candidates’ (Bentley, 1926; Taft, 1933). The Charter Committee elected six of its nominees to the council; the Republican organization, only three.”

Over a very long time, the unsuccessful vote management strategy of the Republican Party during the Cincinnati City Council elections in 1925 was used as an evidence that vote management does not work in the USA. However, Shaw (1954) describes the vote management strategy of the Democratic Party during the New York City Council elections in 1937–1945 as follows:

> “After watching their candidates compete against each other in borough-wide elections in 1937, they evolved their own technique for obtaining maximum representation. Each borough was divided into the same number of zones as the number of councilmen it seemed likely to select. Within each zone the district leaders agreed upon a candidate. Then the entire slate was reviewed by the County Leader and Executive Committee, who ordered the party’s adherents to follow an identical pattern of voting — i.e. the number to be placed beside each candidate’s name in each zone was determined in advance. Proportionalists and their opponents agree that under this system in the 1939 election the Democrats massed their strength for optimum effectiveness.”

With this strategy, the Democratic Party usually won about two thirds of the seats of the New York City Council with about half of the first preferences (Hallett, 1941, 1946; Hermens, 1968; McCaffrey, 1939; Shaw, 1954; Zeller, 1947).

4.2. Equalizing of First Preferences and Later Preferences

A party that wants to run a vote management strategy successfully is confronted with five problems. The first problem is: How should the voters in the different bailiwicks vote?

Equalizing the First Preferences: Suppose that the party tries to win 6 seats. Then this party will divide the electorate into 6 bailiwicks and nominate 6 candidates *a*, *b*, *c*, *d*, *e*, and *f*. When it is sufficient to equalize the numbers of first preferences, then the party will simply ask all its supporters in the first bailiwick to give their first preferences to candidate *a*, all its supporters in the second bailiwick to give their first preferences to candidate *b*, all its supporters in the third bailiwick to give their first preferences to candidate *c*, etc..

Equalizing the Permutations: When also the numbers of the lower preferences have to be equalized, then the optimal strategy would be to divide the electorate into as many bailiwicks as there are permutations of candidates (here: 6! = 720) and to allocate to each bailiwick one of these permutations and to ask all the supporters of this bailiwick to rank the candidates according to this permutation. However, such a strategy would be impracticable; and simply asking the supporters to rank the candidates in a random manner would rather make the voters rank the candidates in that permutation in which the candidates are listed on the ballot than lead to an even distribution of the possible permutations. A practicable simplification of this strategy is to divide the electorate into 6 bailiwicks and to ask all the supporters in the first bailiwick to vote $a \succ_v b \succ_v c \succ_v d \succ_v e \succ_v f$, all the supporters in the second bailiwick to vote $b \succ_v c \succ_v d \succ_v e \succ_v f \succ_v a$, all the supporters in the third bailiwick to vote $c \succ_v d \succ_v e \succ_v f \succ_v a \succ_v b$, etc..

Grouping: Another practicable simplification is to group the candidates and to divide the electorate into as many bailiwicks as there are groups of candidates. Then each bailiwick is divided into as many sub-bailiwicks as there are permutations of the candidates of this group. For example, when the candidates are divided into two groups *abc* and *def* then each bailiwick is divided into 3! = 6 sub-bailiwicks; in these 12 sub-bailiwicks the voters are asked to vote $a \succ_v b \succ_v c \succ_v ...$, $a \succ_v c \succ_v b \succ_v ...$, $b \succ_v a \succ_v c \succ_v ...$, $b \succ_v c \succ_v a \succ_v ...$, $c \succ_v a \succ_v b \succ_v ...$, $c \succ_v b \succ_v a \succ_v ...$, $d \succ_v e \succ_v f \succ_v ...$, $d \succ_v f \succ_v e \succ_v ...$, $e \succ_v d \succ_v f \succ_v ...$, $e \succ_v f \succ_v d \succ_v ...$, $f \succ_v d \succ_v e \succ_v ...$ resp. $f \succ_v e \succ_v d \succ_v ...$. The vote management strategy of the Republican Party during the Cincinnati City Council elections in 1925 is an example of such a grouping strategy.

The equalization of the permutations becomes the more important and simultaneously the more impracticable the larger the number of seats per constituency gets. Therefore, in Ireland, where the electorate is usually divided into 3- to 5-seat constituencies, a party usually only tries to equalize the numbers of first preferences of its candidates, while in the USA, where the electorate is usually divided into 7- to 9-seat constituencies, a party has to find a compromise in this trade-off. Grouping the candidates can be such a compromise.

4.3. Number of Candidates

The second problem is: How many candidates should be nominated?

Suppose C_0 is the number of candidates nominated by that party that runs a vote management strategy. C_0 must not be chosen too large (compared to the number of seats this party could realistically win) because otherwise the candidates are too far below the Droop quota $N/(M+1)$ and eliminated early

(Cohan, 1975, 1978; Lijphart, 1979). When C_0 is chosen too large then there is also the danger that the candidates do not participate at the vote management strategy, since the probability that they will belong to the elected candidates is low; these candidates will rather spend most of their time and money to fight against candidates of their own party (Bax, 1973; Bowler, 1991b; Mair, 1987a; Yates, 1990).

On the other side, when C_0 is chosen too pessimistically then it can happen (a) that the party wastes seats by nominating fewer candidates than it could have won seats, (b) that politicians who have not been nominated, but who believe that the party could possibly win more seats than it has nominated candidates, run as independent candidates (Busteed, 1990; Holmes, 1999) or (c) that the party is punished by the voters of a certain minority for not having nominated politicians of this minority (Gallagher, 1980; Mair, 1987a). A large number of candidates also leads to a higher turnout among the supporters of this party; the larger the number of candidates is the larger is the number of supporters who feel represented by at least one candidate and who, therefore, go to the polls and cast a vote that will — as soon as their favorite candidate is eliminated — be transferred to the front-runners of this party (Gallagher, 1980, 1988a; Goldman, 1930; Katz, 1981; Mair, 1987a, 1987b; Marsh, 2000). Furthermore, many voters give their first preference to their favorite candidate (*primacy of locality over party*) and their lower preferences to the party of their favorite candidate (*primacy of party over locality*) even when their favorite candidate has no chance to be elected (Bowler, 1991a; Busteed, 1990; Gallagher, 1980, 1988a; Garvin, 1976; Katz, 1981; Mair, 1986, 1987a, 1987b; Marsh, 2000); in extreme cases, this voting behaviour makes a party nominate more candidates than there are seats in this constituency (Mair, 1986).

A larger number of candidates is also needed when vacant seats are filled by *recounts* (That means: When a seat gets vacant then this seat is filled by counting the ballots of the last regular elections anew, but with the current members of parliament "immune to elimination" and with the ineligible candidates ignored.) because otherwise there is the danger that this party loses a seat to another party when this seat gets vacant and is filled by a recount; the common tactic is to nominate to each candidate who gets a bailiwick a sufficiently large number of *running mates*.

Example: A party uses vote management to win X = 5 seats. The party believes that the same seat will get vacant not more than Y = 2 times. Then this party will nominate X·(Y+1) = 15 candidates (= 5 candidates a_1, b_1, c_1, d_1, e_1 and 10 running mates a_2, b_2, c_2, d_2, e_2, a_3, b_3, c_3, d_3, e_3). It will divide the district into 5 bailiwicks. In the first bailiwick, it will ask its supporters to vote $a_1 \succ_v a_2 \succ_v a_3 \succ_v b_1 \succ_v b_2 \succ_v b_3 \succ_v c_1 \succ_v c_2 \succ_v c_3 \succ_v d_1 \succ_v d_2 \succ_v d_3 \succ_v e_1 \succ_v e_2 \succ_v e_3$; in the second bailiwick, it will ask its supporters to vote $b_1 \succ_v b_2 \succ_v b_3 \succ_v c_1 \succ_v c_2 \succ_v c_3 \succ_v d_1 \succ_v d_2 \succ_v d_3 \succ_v e_1 \succ_v e_2 \succ_v e_3 \succ_v a_1 \succ_v a_2 \succ_v a_3$; in the third bailiwick, it will ask its supporters to vote $c_1 \succ_v c_2 \succ_v c_3 \succ_v d_1 \succ_v d_2 \succ_v d_3 \succ_v e_1 \succ_v e_2 \succ_v e_3 \succ_v a_1 \succ_v a_2 \succ_v a_3 \succ_v b_1 \succ_v b_2 \succ_v b_3$; in the fourth bailiwick, it will ask its supporters to vote $d_1 \succ_v d_2 \succ_v d_3 \succ_v e_1 \succ_v e_2 \succ_v e_3 \succ_v a_1 \succ_v a_2 \succ_v a_3 \succ_v b_1 \succ_v b_2 \succ_v b_3 \succ_v c_1 \succ_v c_2 \succ_v c_3$; and in the fifth bailiwick, it will ask its supporters to vote $e_1 \succ_v e_2 \succ_v e_3 \succ_v a_1 \succ_v a_2 \succ_v a_3 \succ_v b_1 \succ_v b_2 \succ_v b_3 \succ_v c_1 \succ_v c_2 \succ_v c_3 \succ_v d_1 \succ_v d_2 \succ_v d_3$. Advantages of this strategy are: (1) A direct confrontation between a candidate and his running mates is prevented. (2) When a representative is replaced by his running mate, then it is still clear which representative represents which bailiwick, so that there is no need to redraw the bailiwicks for the next regular elections.

Thus, STV suffers from the same overnomination and undernomination problems as cumulative voting, limited voting, and SNTV. However, these problems are less severe under STV than under other multi-winner election methods since Droop proportionality guarantees a minimum representation for each party.

4.4. Criteria

The third problem is: According to which criteria should the electorate be divided?

Usually the electorate is divided according to geographic or social criteria. However, sometimes the electorate is divided according to the ID numbers. Example: In 1997, two organizations, the *Specialists* and the *Initiative*, ran for the two students' seats of the Communication and Historical Sciences Department of the Berlin University of Technology. The Specialists ran a single list. The Initiative ran two lists; its first list was named “Even List” and its second list was named “Odd List”. In its newspaper and during campaign speeches, the Initiative asked its supporters to vote for the Even List when their matriculate number was even and for the Odd List when their matriculate number was odd. The reason for this strategy was that this council was elected by the largest remainder method of proportional representation by party lists; if the Initiative had run only one list then it would have had to get more than three fourths of the votes to win both seats; however, with two lists and a sufficiently even distribution of its votes the Initiative needed only more than two thirds of the votes to win both seats. In the end, the Even List got 129 votes and the Odd List got 110 votes; with this vote management strategy the Initiative won both students' seats. When $N = 239$ votes are distributed in a random manner x:y (here x = y = 1), then the standard deviation is $(\sqrt{(N \cdot x \cdot y)})/(x+y) = 8$. Therefore, the division of the supporters was as even as statistically possible.

In Taiwan, the electorate is sometimes divided according to the birthdays. Liu (1999) writes about the SNTV elections in Taiwan:

> “The two opposition parties in Taiwan, the Democratic Progressive Party (DPP) and the two-year-old New Party (NP), adopted a new device to allocate their potential votes among their nominees in the election of 1995. By placing advertisements in newspapers and making announcements during campaign speeches, both parties asked their supporters to cast votes according to their birth months. Since the DPP nominated four candidates in the South District of Taipei City, the DPP divided voters into four groups: those who were born in January, February, and March; those who were born in April, May, and June; and so on. Each group was asked to vote for one candidate. The NP did the same thing. It nominated six candidates in the two districts in Taipei, three in each. It accordingly divided the voters into three groups using the same approach as the DPP. The aim of this tactic was to allocate the party's voters evenly among its nominees. Both parties evidently assumed that the identification of their supporters with each party was strong enough to induce them to vote as the party instructed. The final results support this speculation.”

In Taipei South, the DPP got $N = 211{,}559$ votes; with x = 1 and y = 3 the standard deviation is 199; however, the candidates got 56,848, 56,364, 50,072 resp. 48,275 votes. In Taipei North, the NP got $N = 171{,}667$ votes; with x = 1 and y = 2 the standard deviation is 195; however, the candidates got 61,259, 59,575 resp. 50,833 votes. In Taipei South, the NP got N = 167,938 votes; with x = 1 and y = 2 the standard deviation is 193; however, the candidates got 60,485, 54,456 resp. 52,997 votes. Therefore, the deviations are significantly larger than expected for a purely random distribution. Possible explanations are the different lengths of the months and perhaps an uneven distribution of the birthdays over the calendar days. However, the most probable explanation is that the candidates were differently popular; deviations caused by different popularities can be circumvented by using *asymmetric vote management* (section 4.6).

There is the tendency that candidates get more votes when they are listed higher on the ballot, even when the candidates are listed in an alphabetic manner or in a random manner on the ballot (*alphabetic voting*; *donkey voting*) (Bakker, 1980; Chen, 1994; Darcy, 1990, 1993; Hermens, 1968; Kelley, 1984; Kestelman, 2002; Robson, 1974; Sowers, 1934). To minimize this so-called *ballot position effect* some countries use *Robson rotation*. That means: Starting with a ballot on which the candidates are listed in an alphabetic manner, the list of candidates on the other ballots is changed in a cyclic manner so that each candidate is on the top of the same number of ballots; ballots with the same candidate on top are distributed evenly over the constituency (Hughes, 2000). In Tasmania, the candidates are listed on the ballot according to their party label; candidates with the same party label are listed in an alphabetic manner; among candidates with the same party label Robson rotation is being used. Therefore, the electorate can be divided into bailiwicks simply by asking the voters to rank the candidates in the same manner in which they are listed on the ballot. This vote management strategy has been predicted; but it has not yet been observed (Farrell, 1996, 2000, 2006).

When the electorate is sufficiently small (e.g. when the upper house is elected by an electoral college using STV), then the electorate can be divided into bailiwicks in a completely arbitrary manner. For example, Braunias (1932) and Sternberger (1969) write about the indirect elections to the upper house (*Landsting*) of Denmark in 1866–1953:

> "The seats are distributed according to Andrae's method. However, this method is not used in practice. The parties rather divide their electors into groups so that each group contains a quota of members. Each group writes on their ballots the name of only one Landsting member and the names of three substitutes."

The above statement ("The seats are distributed according to Andrae's method. However, this method is not used in practice.") does not make much sense. I guess that Braunias (1932) and Sternberger (1969) want to say that no transfer of votes occurs. However, it is necessary to keep in mind that the Andrae method does not transfer votes of eliminated candidates and it transfers surpluses only when the elected candidate has more than an Andrae-Hare quota N/M of votes. As a candidate is certain to be elected already when he has more than $N/(M+1)$ votes (even when the Andrae-Hare quota is being used), the use of the Andrae-Hare quota in Denmark made STV very prone to free riding strategies and vote management strategies.

In most cases, more than one criterion is used to divide the electorate into bailiwicks. For example, Sacks (1970) writes that, in the Donegal North-East constituency, Fine Gael gave its candidate Bertie Boggs not only a geographic part of the constituency but also the Protestant voters of the other parts of this constituency. Winters (2001) and Pitkin (2001) write that in Cambridge, Massachusetts, the voters are always divided according to a wide range of criteria simultaneously (Winters: "mix of issue-based, geography-based, and identity-based"; Pitkin: "ideological, geographic, ethnic, or familiar factions").

4.5. Strictness

The fourth problem is: How strict should the division into bailiwicks be? Marsh (2000) writes:

> "Once the candidates are selected, the issue of running their individual campaigns to best joint advantage is likely to cause some difficulty. There are three ways to handle this. The first is the 'anarchy' option: do nothing, and allow each candidate to campaign across the whole constituency and let matters take their course. Conflict often arises between candidates competing for the same first preference votes. This can be handled by the second option: 'bailiwicking'. Here, each candidate is allocated a particular area of the constituency within which their campaign should be concentrated. Deals may be made about the extent to which any candidate is allowed, on certain days, out of a particular 'bailiwick' but essentially the solution consists of an agreement (or injunction) for the candidates to stay apart from one another. In many cases a weak 'bailiwick' system is agreed where the candidates agree to stay out of each other's 'home' areas but the remainder of the constituency is open to all. Such arrangements may avoid conflict, or at least constrain it, but it does not necessarily lead to an efficient outcome for the national party. The system may produce a fairly even division of votes between a party's candidates (Farrell, 1996) but variations in the strength of a party's candidates, and the strength of the opposition's candidates make it very uncertain. The third solution is direct 'vote management'. Here the objective is to divide the vote up between the candidates so as to maximise the number of seats obtained. The need, or opportunity for such a strategy is the fact that preferences can only be transferred to a candidate who remains in the race. A party's candidates may sometimes win enough votes to expect two seats but fail to get them because too many votes go to one candidate and the second is eliminated before those votes can be transferred, or finishes as runner-up when the first candidate has votes to spare. Equalising votes between candidates is a difficult operation. It requires a fairly accurate assessment of the party's overall voting strength, both in terms of first and later preferences. (Local polling has been used, particularly in Fine Gael, to provide information in this respect.) It requires a set of voters who are willing to vote for the candidate that they are advised by the local party to vote for. And it requires the stronger candidate to give up votes by advising supporters to vote for a running mate — and risk defeat in the process. It does happen. In a notable case former Fine Gael party leader Garret FitzGerald succeeded in winning two seats in his constituency in 1989 by advising many of his supporters to vote for his running mate, on the assumption that FitzGerald could pick up transfers from all parties. The ploy succeeded, but only just. Gallagher (1993a) estimated that in 1992 Fianna Fáil could have won nine more seats with perfect vote management, Fine Gael two and Labour one (plus a couple more where it should have run more candidates), and similar tales could be told about missed opportunities

> in other elections. Generally it seems such management can be undertaken only with the active support of the stronger candidate(s). National party executives lack the authority to impose it but Farrell (1993) argues the practice has become more widespread in recent years."

Bowler (1991b) writes:

> "In response to these pressures candidates from the same party may draw up an agreement which restricts campaigning for first preferences to geographically restricted areas. These contracts vary in both specificity and formality. In the Wexford constituency in 1989, for example, the Fine Gael candidates drew up a formal Code of Conduct which divided up the constituency into areas over which each candidate had sole right of campaigning. Each Fine Gael candidate in Wexford was also allotted two days in which to campaign across the entire constituency. This type of campaigning will overlap with, and may even be the main basis of, 'friends and neighbours' effects. The point here is that it also provides incentives for candidates from within the same party to cheat on such agreements and so engage in a campaign against fellow party members. Candidates from within the same party, then, can be seen to contribute to the usual fissiparous effect of PR upon party unity and stability. The importance of intra-party competition in terms of voter choice is seen when we note that in 1982, for instance, 17 of the 25 sitting Fianna Fáil deputies who lost did so to members of their own party; for Fine Gael, this figure was 5 out of 12."

4.6. Symmetry

The fifth problem is: Should the candidates get bailiwicks of same size or of different size?

When the candidates get bailiwicks of same size, then this is called *symmetric vote management*; otherwise, this is called *asymmetric vote management*. There are mainly three reasons why candidates should get bailiwicks of different size. First: In Ireland, the constitution says (1) that the Prime Minister (*Taoiseach*), the Deputy Prime Minister (*Tánaiste*), and the Minister of Finance have to be members of the lower house, (2) that all other Cabinet Ministers have to be members of the lower house (*Dáil*) or the upper house (*Seanad*), and (3) that not more than two Cabinet Ministers can be members of the Seanad; however, by tradition, all Cabinet Ministers and all Junior Ministers have to be members of the Dáil. In Malta, the constitution says that all Cabinet Ministers have to be members of the unicameral parliament. Therefore, it is necessary to make the election of those about 30 candidates (Ireland) resp. those about 15 candidates (Malta) who are designated for government offices certain; as the coalition usually has only about 80 seats (Ireland) resp. about 35 seats (Malta) and as STV makes every seat vulnerable, it is very difficult to make the election of a given candidate certain. Because of these regulations resp. because of similar regulations in other countries (For example: In some communities in the USA, it was tradition that that candidate, who has been elected at the earliest stage of the count resp. — when more than one candidate has been elected at this stage — of all those candidates who have been elected at this stage that candidate who has got the largest surplus at this stage, becomes mayor (Amy, 1993).), those candidates, who are designated for government offices, usually get larger bailiwicks.

However, also the opposite tendency can be observed: As these regulations make voters vote preferably for candidates who are designated for government offices (because voters have an interest that government officials come from their constituency and because these regulations mean that these candidates necessarily have to be elected into the parliament to be able to get government offices), such candidates need only a smaller bailiwick. This voting behaviour is strengthened by the fact that, in the ideal case, an additional vote for candidate *a* of party X helps party X win an additional seat only when candidate *a* gets this additional seat. This voting behaviour can best be observed in those constituencies where a given party cannot hope to win more than one seat, because here this voting behaviour cannot be attributed to asymmetric vote management. Busteed (1990) writes:

> “It has already been noted how in June 1981 North Kerry elected the Labour TD Dick Spring, but he only took the seat on the sixth and last count with the narrow margin of 144 votes over his Fine Gael opponent. Subsequently he was appointed Minister of State at the Department of Justice, and his electoral position was transformed. In the February 1982 general election he topped the poll and was elected on the first count with 25.8% of first preferences. Recent injury in a car crash may also have pulled in sympathy votes. By November 1982 he was Labour leader and clearly destined to be Tánaiste in any future coalition. The prospect improved his position still further: again he topped the poll, this time with 28.9% of first preferences. The explanation for his success may have lain in pride at a local boy made good, but it seems equally likely that the electors of North Kerry were drawn to him by the expectation that from his increasingly powerful position he could divert a good deal of patronage to the constituency and by their votes they simultaneously strengthened his base and established a claim on him. (...)
>
> The belief is that the promotion of a TD will in a broad sense flatter constituency pride. Voters for their party will both bask in reflected glory and expect that their representatives’ extra pull will now be deployed to the community’s material advantage when it comes to the allocation of public resources. Such considerations are widely believed to be particularly influential in the appointment of the junior ministers known as Ministers of State. Such posts it is suggested are sometimes allocated to TDs who won marginal seats in the hope that this will consolidate their positions at the next election. For example, in the 1981 coalition government Michael Begley, Fine Gael TD for Kerry South was appointed Minister of State for Trade, Commerce, and Tourism. He was the only party deputy in either Kerry constituency and his seat seemed vulnerable since his first preference vote had fallen by over 500 compared with the 1977 election and he had only been elected at the fourth count. If his appointment was designed to strengthen his position it succeeded since he was re-elected comfortably in both February and November 1982, his share of first preferences rising from 18.3% through 20.8% to 22.8%.”

<u>Second:</u> Especially at universities and in private organizations, but also in public elections, frequently the bylaws require that each social group must be represented by a minimum number of seats. This is called “STV with constraints” (Hill, 1998, 1999; Kitchener, 1999; Otten, 2001). For example, the constitution of Edmonton, Alberta, said that at least 3 of the 7 members of the City Council have to come from the Strathcona borough (Hoag, 1926; Johnston, 2000). Such a regulation is fulfilled by declaring a candidate

“immune to elimination” when his elimination would lead to a violation of this regulation. The consequence of such a regulation is that in a vote management strategy candidates who are believed to become “immune to elimination” get a bailiwick which is only as large as necessary to guarantee that these candidates still become “immune”.

The same phenomenon occurs when all voters of a given minority strictly prefer each candidate of this minority to each other candidate, independent of the party labels of the candidates (*religious voting*; *racial voting*). For example, Protestant voters usually strictly prefer each Protestant candidate to each other candidate (Busch, 1995; Gallagher, 1980; Moley, 1918; Sacks, 1970); the same is valid for Afro-Americans (Burnham, 1990, 1997; Goldman, 1930; Gosnell, 1930; Harris, 1930), Poles (Anderson, 1995; Moley, 1923), Italians (Busch, 1995; Moscow, 1967), etc.. When it is clear in advance that a given minority will win M_p seats, then it is sufficient that the candidates of this minority get only bailiwicks of $N_p/(M_p+1)$ voters, where N_p is the number of voters of this minority. When e.g. N_p is slightly larger than a Droop quota $N/(M+1)$, then it is sufficient that the bailiwick of the candidate of this minority is slightly larger than a half Droop quota; at each stage of the count, this candidate of the minority will have more votes than every other candidate of this minority so that he will get the votes of the other candidates.

Third: As popular candidates usually win more additional votes during the transfer of votes than unpopular candidates do, popular candidates sometimes get smaller bailiwicks. In the above quotation by Marsh (2000), Garret FitzGerald did not get any bailiwick at all since his party, the Fine Gael, expected that he will win so many additional votes during the count that he did not need any bailiwick at all. With this strategy, the Fine Gael, which had won only one of the 4 seats of the Dublin South-East constituency with 32.0% of the first preferences in 1987 (FitzGerald: 21.1%; Joseph Doyle: 8.7%; William Egan: 2.2%), won 2 of the 4 seats with 27.7% in 1989 (Doyle: 15.9%; FitzGerald: 11.7%) (Gallagher, 1990a). On the other side, as sometimes popular candidates are willing to participate at a vote management strategy only when their re-election is not put in danger by this strategy, these candidates sometimes get a larger bailiwick (Marsh, 2000). Bailiwicks of different size are also necessary when the strengths of the opposition’s candidates differ too much (Marsh, 2000). In the end, the size of the bailiwick of a given candidate depends mainly on his interests and on his ability to get his way (Mair, 1987a, 1987b).

Example: In 1973, the Fianna Fáil won only one of the 3 seats in the Sligo-Leitrim constituency with 50.6% of the first preferences (Raymond McSharry: 25.8%; Bernard M. Brennan: 16.9%; Joseph Mary Mooney: 8.0%). To win 2 seats in 1977, Fianna Fáil decided to use vote management. As McSharry was willing to participate at this vote management only when this did not risk his re-election, this constituency was not divided 1:1 but 2:1, where McSharry got the larger part and James Gallagher got the smaller part. Although the vote share of the Fianna Fáil dropped from 50.6% in 1973 to 48.0% in 1977, it could win 2 of the 3 seats with this vote management. Mair (1986) writes:

> “A good example of such a division of territory is afforded by the Sligo-Leitrim constituency in 1977, where Fianna Fáil was attempting to win two seats in a very marginal 3-seat constituency. The constituency was dominated by one major urban center, Sligo town, and was otherwise a predominantly rural area. One of the two Fianna Fáil candidates (McSharry) based his support in Sligo town, while the

> other (Gallagher) was based southern end of the county, a mainly farming area with small market towns such as Ballymote, Colooney, and Tubbercurry (Gallagher’s home town). In this particular case, the bailiwicking was easily effected: McSharry had the town and coastal strip, or what he referred to as the area ‘from Bundoran to Ballina, between the mountains and the sea,’ while Gallagher had the remaining and admittedly smaller part of the constituency. In the former area the Fianna Fáil posters urged supporters to ‘vote No. 1 McSharry and No. 2 Gallagher,’ and in the latter ‘vote No. 1 Gallagher and No. 2 McSharry.’ A border was clearly delineated by both candidates, and so the contest was organized. In the event, Fianna Fáil did win two of the three seats with an overall vote of 48.0% (30.8% for McSharry and just 17.3% for Gallagher), and with over 72% of McSharry’s surplus passing to Gallagher in the second count.”

Of course, the above mentioned vote management strategies can be combined in many different manners. For example, Galligan (1999) writes that in the elections to the Dáil in 1997 the Fianna Fáil divided the 4-seat Longford-Roscommon constituency 2:3 where the smaller part went to the TD and former Taoiseach Albert Reynolds while the larger part went to the candidates Seán Doherty, Michael Finneran, and Terry Leyden. This is not only an example for asymmetric vote management (as the constituency was divided 2:3 and not 1:3), but also for differently strict divisions. In these elections, Reynolds got 39.3%, Doherty 25.9%, Finneran 19.9%, and Leyden 14.9% of all first preferences of the Fianna Fáil. Reynolds and Doherty were elected while Finneran and Leyden were eliminated. With 47.0% of all first preferences the Fianna Fáil won 2 of the 4 seats; therefore, it did not win more seats than it would have won without this vote management strategy; however, this vote management strategy guaranteed that Reynolds was among the elected Fianna Fáil candidates.

4.7. Summary

Vote management is a strategy where a party or a group of independent candidates asks its supporters to vote preferably for those of its candidates who are less assured of election.

Vote management is neither an invention of the 1970s or 1980s nor a pure Irish phenomenon. The political organizations rather understood the usefulness of vote management almost immediately. But it always took very much experience until they were able to run vote management successfully. Especially, it took very much time until they observed that it is usually not useful to divide the electorate into bailiwicks of same size.

Today, its vulnerability to vote management strategies is STV’s most serious problem. The Royal Commission on the Electoral System (New Zealand 1986), the Plant Commission (UK 1993), and the Jenkins Commission (UK 1998) rejected STV partly because of Ireland’s bad experience with vote management. Vote management also has a bad influence on intra-party democracy: Frequently, candidates complain that their party did not allow them to campaign in the whole constituency (McDaid, 1993; Sacks, 1970). Other authors call vote management “corrupting” (Kestelman, 2000) and compare the Irish parties with the “party machines” (Bax, 1973) in the USA or even with the “Mafia” (Sacks, 1976). Therefore, each attempt to find better STV methods should primarily try to reduce its vulnerability to vote management.

5. Proportional Completion

In sections 6 and 7, we will restrict our considerations to situations where each voter $v \in V$ casts a linear order $\succ_v$ on A. Therefore, we have to explain how non-linear individual orders are completed to linear individual orders when some voters cast non-linear orders.

The common way to complete non-linear individual orders to linear individual orders is symmetric completion. *Symmetric completion* means: When voter $v \in V$ is indifferent between the candidates $f_1,...,f_n \in A$, then this voter is removed from V and, for each of the $n!$ possible permutations $\{\sigma(1),...,\sigma(n)\}$ of $\{1,...,n\}$, a voter u is added to V who has the weight $1/(n!)$, who ranks the candidates $f_{\sigma(1)} \succ_u ... \succ_u f_{\sigma(n)}$, and who ranks the other candidates relatively to each other in the same manner as voter v did.

However, a problem with symmetric completion is that adding an empty ballot and completing this ballot in a symmetric manner can change the result of the elections. Also adding a voter, who prefers a very weak candidate (resp. a very strong candidate) to every other candidate and who is indifferent between every other candidate, and completing his ballot in a symmetric manner can change the result of the elections. One would expect that adding a voter, who is indifferent between all those candidates whose election is unclear, doesn’t change the result of the elections.

Because of these reasons, we propose proportional completion as an alternative to symmetric completion. *Proportional completion* means that non-linear individual orders are completed to linear orders in such a manner that, for each set of candidates, the proportions of the (to these candidates restricted) individual orders are not changed. So when N_{empty} voters, who are all indifferent between all those candidates $f_1,...,f_n \in A$ whose election is unclear, are added, then proportional completion simply means that, for each linear order of the candidates $f_1,...,f_n \in A$, the number of ballots with this order is multiplied by $1 + N_{empty}/N_{old}$.

Example 1: Suppose a voter is indifferent between candidate a and candidate b. Suppose of the other voters $X_1 = 56$ strictly prefer candidate a to candidate b and $X_2 = 44$ strictly prefer candidate b to candidate a, then this voter is replaced by $X_1/(X_1+X_2) = 0.56$ voters who rank these candidates $a \succ_v b$ and by $X_2/(X_1+X_2) = 0.44$ voters who rank these candidates $b \succ_v a$ and who rank the other candidates in the same manner as the original voter did.

Example 2: Suppose a voter is indifferent between the candidates a, b, c, and d. Suppose $X > 0$ voters are not indifferent between these candidates. Suppose X_1 voters rank these candidates $a \approx_v c \succ_v b \approx_v d$, X_2 voters rank them $d \succ_v a \approx_v b \approx_v c$, X_3 voters rank them $c \approx_v d \succ_v b \succ_v a$, etc.. Then this voter is replaced by X_1/X voters who rank them $a \approx_v c \succ_v b \approx_v d$, X_2/X voters who rank them $d \succ_v a \approx_v b \approx_v c$, X_3/X voters who rank them $c \approx_v d \succ_v b \succ_v a$, etc. and who rank the other candidates in the same manner as the original voter did. At additional stages, these still incomplete individual orders are further completed.

5.1. Definition of Proportional Completion

The following 3 stages give a precise definition for proportional completion.

Stage 1:

W shall be the proportional completion of V. $\rho(w) \in \mathbb{R}$ shall be the weight of voter $w \in W$. Then we start with

(a) $W := V$.

(b) $\forall\, w \in W$: $\rho(w) := 1$.

Stage 2:

Suppose there is a voter $w \in W$ and a set of candidates $f_1,...,f_n \in A$ with

(a) $n > 1$.

(b) $\forall\, f_i,f_j \in \{f_1,...,f_n\}$: $f_i \approx_w f_j$.

(c) $\forall\, f_i \in \{f_1,...,f_n\}\ \forall\, e \in A \setminus \{f_1,...,f_n\}$: $f_i \not\approx_w e$.

Suppose $X \in \mathbb{N}_0$ is the number of voters $v \in V$ with

(5.1.1) $\exists\, f_i,f_j \in \{f_1,...,f_n\}$: $f_i \not\approx_v f_j$.

Case 1: $X > 0$.

For each voter $v \in V$ with (5.1.1), a voter u is added to W with

(5.1.2) $\forall\, g,h \in A \setminus \{f_1,...,f_n\}$: $g \succ_w h \Leftrightarrow g \succ_u h$.

(5.1.3) $\forall\, f_i \in \{f_1,...,f_n\}\ \forall\, g \in A \setminus \{f_1,...,f_n\}$: $g \succ_w f_i \Leftrightarrow g \succ_u f_i$.

(5.1.4) $\forall\, f_i \in \{f_1,...,f_n\}\ \forall\, h \in A \setminus \{f_1,...,f_n\}$: $f_i \succ_w h \Leftrightarrow f_i \succ_u h$.

(5.1.5) $\forall\, f_i,f_j \in \{f_1,...,f_n\}$: $f_i \succ_v f_j \Leftrightarrow f_i \succ_u f_j$.

(5.1.6) $\rho(u) := \rho(w) / X$.

Case 2: $X = 0$.

For each of the $n!$ possible permutations $\{\sigma(1),...,\sigma(n)\}$ of $\{1,...,n\}$, a voter u is added to W with (5.1.2) – (5.1.4) and

(5.1.7) $\forall\, f_i,f_j \in \{f_1,...,f_n\}$: $\sigma(i) > \sigma(j) \Leftrightarrow f_i \succ_u f_j$.

(5.1.8) $\rho(u) := \rho(w) / (n!)$.

After all these voters u have been added to W, the original voter w is removed from W.

Stage 3:

Stage 2 is repeated until $a \not\approx_w b\ \forall\, a \in A\ \forall\, b \in A \setminus \{a\}\ \forall\, w \in W$.

6. A New STV Method

In sections 3 and 4, we saw that today Hylland free riding and vote management are the two most serious problems of STV methods. In this section, we introduce a mathematical model to describe vote management (section 6.1). In section 6.2, we use this model to design an STV method that is vulnerable to Hylland free riding and vote management only in those cases in which otherwise Droop proportionality would have to be violated.

6.1. Equivalence of Free Riding and Vote Management

In sections 3 and 4, we explained why today Hylland free riding and vote management are the two most serious problems of STV methods. Nevertheless, we are aware of only a few papers that try to explain which property of STV methods is that property that is misused in a vote management strategy. For example, Hylland (1992) writes:

> "Since candidates can be elected with fewer votes than the quota, there are cases in which a group can gain by dividing the first preferences of its supporters evenly among its candidates, securing each of them fewer than the quota, but enough to be elected."

Farrell (2006) writes:

> "It is generally argued in the literature that parties aiming to maximise the number of seats they win should try to spread the preferences as evenly as possible among their candidates, to give all their candidates a good chance of remaining in the count long enough to pick up preferences from other parties. An inappropriate distribution of the preference vote between candidates of the same party can lead to that party losing an otherwise winnable seat. As Cox (1997) put it, the parties should try to redirect votes from 'vote-rich' to 'vote-poorer' candidates. According to Gallagher (1992): 'Under the assumption that no votes transfer across party lines, a party's ideal strategy under STV is to ensure that all of its candidates have exactly the same number of votes at every stage of the count, and that when any one is eliminated, his or her votes transfer evenly among the party's other candidates.'"

According to Marsh (2000), a vote management strategy misuses the fact that "preferences can only be transferred to a candidate who remains in the race" and according to Hylland a vote management strategy misuses the fact that, because of ballots that have become exhausted, "candidates can be elected with fewer votes than the quota". Marsh's and Hylland's conjectures are problematic because of two reasons. First: The fact that preferences cannot be transferred to already eliminated candidates and the fact that candidates can be declared elected without reaching the quota are rather parts of the *description of the counting process* of traditional STV methods than a *property* of these STV methods. In other words, whether a given STV method is vulnerable to vote management strategies must not depend on whether this STV method is defined in an *axiomatic* manner or an *algorithmic* manner. Second: Their conjectures cannot explain why in multi-winner elections spreading the preferences evenly among the party candidates is a useful strategy even when no elimination of candidates and, therefore, also no exhaustion of ballots occur.

Example 1 (N = 100 voters; M = 2 seats; C = 3 candidates):	
10 voters	$a \succ_v b \succ_v c$
35 voters	$a \succ_v c \succ_v b$
25 voters	$b \succ_v c \succ_v a$
30 voters	$c \succ_v b \succ_v a$

In example 1, the winners are the candidates *a* and *c*. However, when the candidates *a* and *b* run a vote management strategy against candidate *c* and ask their supporters to vote preferably for candidate *b* then this example looks as follows:

Example 2 (N = 100 voters; M = 2 seats; C = 3 candidates):	
10 voters	$b \succ_v a \succ_v c$
35 voters	$a \succ_v c \succ_v b$
25 voters	$b \succ_v c \succ_v a$
30 voters	$c \succ_v b \succ_v a$

Now, the winners are the candidates *a* and *b*. The fact that vote management is possible although only 3 candidates are running for 2 seats so that no elimination of candidates and, therefore, also no exhaustion of ballots occur demonstrates that none of these properties can be that property that is misused in a vote management strategy.

This example demonstrates that also (1) the special rules to transfer surpluses or (2) the violation of monotonicity cannot be that property of STV methods that is misused in a vote management strategy.

Presumption of this paper is that the vulnerability to Hylland free riding is that property of STV methods that is misused in a vote management strategy. To be more concrete: We presume that the term "Hylland free riding" and the term "vote management" refer to the same strategic problem. The only difference is that the term "Hylland free riding" refers to this strategic problem seen from the point of view of an individual voter who tries to maximize the influence of his vote by voting preferably for those of his favorite candidates who are less assured of election, while the term "vote management" refers to a political party or a group of independent candidates that tries to maximize its number of seats by asking its supporters to vote preferably for those of its candidates who are less assured of election. Consequences of this presumption are (1) that an election method is vulnerable to vote management if and only if it is vulnerable to Hylland free riding and (2) that an election method is the more vulnerable to vote management the more vulnerable it is to Hylland free riding.

The example with Garret FitzGerald (sections 4.5 and 4.6) demonstrates that, for voter *v* to participate at a vote management strategy of the candidates $a_1,...,a_M$ against candidate *b*, it is sufficient that voter *v* prefers that candidate a_k, for which this voter has to vote in this vote management strategy, to candidate *b*. It is not necessary that this voter prefers all the candidates $a_1,...,a_M$ to candidate *b*. Otherwise, only those voters had voted for Joseph Doyle who prefer both, Doyle and FitzGerald, to the other candidates.

Therefore, we get the following general results: When the used multi-winner election method satisfies Droop proportionality and each voter casts a linear order $\succ_v$ on A, then vote management of the candidates $a_1,...,a_M$ against candidate b is possible if and only if there is a $t \in \mathbb{R}^{(N \times M)}$ with properties (6.1.1) – (6.1.4).

(6.1.1) $\forall\, i \in \{1,...,N\}\ \forall\, j \in \{1,...,M\}$: $t_{ij} \geq 0$.

(6.1.2) $\forall\, i \in \{1,...,N\}$: $\sum_{j=1}^{M} t_{ij} \leq 1$.

(6.1.3) $\forall\, i \in \{1,...,N\}\ \forall\, j \in \{1,...,M\}$: $b \succ_i a_j \Rightarrow t_{ij} = 0$.

(6.1.4) $\forall\, j \in \{1,...,M\}$: $\sum_{i=1}^{N} t_{ij} > N/(M+1)$.

The candidates $a_1,...,a_M$ can then ask each voter i to give t_{ij} of his vote to candidate a_j. In so far as (6.1.3) says that t_{ij} can be larger than 0 only when voter i prefers candidate a_j to candidate b, it is guaranteed that voter i will be willing to give t_{ij} of his vote to candidate a_j. (6.1.4) says that each of the candidates $a_1,...,a_M$ will then have more than $N/(M+1)$ votes and will necessarily be elected according to Droop proportionality.

If there is a $t \in \mathbb{R}^{(N \times M)}$ with properties (6.1.1) – (6.1.3) and (6.1.5), then the candidates $a_1,...,a_M$ can run a vote management strategy against candidate b with the aim that candidate b is, at best, tied for election:

(6.1.5) $\forall\, j \in \{1,...,M\}$: $\sum_{i=1}^{N} t_{ij} \geq N/(M+1)$.

A *Condorcet candidate* is a candidate $b \in A$ such that there is <u>no</u> set of candidates $\{a_1,...,a_M\} \in A_M$ such that there is a $t \in \mathbb{R}^{(N \times M)}$ with properties (6.1.1) – (6.1.3) and (6.1.5).

So when we want to minimize the vulnerability to vote management and Hylland free riding, we should insist that, if candidate $b \in A$ is a Condorcet candidate, then candidate b is in every winning set $\mathcal{S}_M$.

6.2. Definition of the Schulze STV Method

In this section, we propose new STV method, where the vulnerability to Hylland free riding and vote management is minimized.

Stage 1:

Proportional completion is used to complete V to W.
Suppose N_W is the number of voters in W.
Suppose $\rho(w) \in \mathbb{R}$ with $\rho(w) > 0$ is the weight of voter $w \in W$.

Stage 2:

A *path* from set $\mathfrak{X} \in A_M$ to set $\mathfrak{Y} \in A_M$ is a sequence of sets $\mathfrak{C}(1),...,\mathfrak{C}(n) \in A_M$ with the following properties:

1. $\mathfrak{X} \equiv \mathfrak{C}(1)$.
2. $\mathfrak{Y} \equiv \mathfrak{C}(n)$.
3. $2 \leq n < \infty$.
4. For all $i = 1,...,(n-1)$: $\mathfrak{C}(i)$ and $\mathfrak{C}(i+1)$ differ in exactly one candidate. That means: $|\ \mathfrak{C}(i) \cap \mathfrak{C}(i+1)\ | = M - 1$ and $|\ \mathfrak{C}(i) \cup \mathfrak{C}(i+1)\ | = M + 1$.

Suppose $a_1,...,a_M,b \in A$. Then $N[\{a_1,...,a_M\},b] \in \mathbb{R}$ is the largest value such that there is a $t \in \mathbb{R}^{(N_W \times M)}$ such that

(6.2.1) $\forall\ i \in \{1,...,N_W\}\ \forall\ j \in \{1,...,M\}$: $t_{ij} \geq 0$.

(6.2.2) $\forall\ i \in \{1,...,N_W\}$: $\sum_{j=1}^{M} t_{ij} \leq \rho(i)$.

(6.2.3) $\forall\ i \in \{1,...,N_W\}\ \forall\ j \in \{1,...,M\}$: $b \succ_i a_j \Rightarrow t_{ij} = 0$.

(6.2.4) $\forall\ j \in \{1,...,M\}$: $\sum_{i=1}^{N_W} t_{ij} \geq N[\{a_1,...,a_M\},b]$.

If $\mathfrak{X},\mathfrak{Y} \in A_M$ differ in exactly one candidate, then we define $\tilde{N}[\mathfrak{X},\mathfrak{Y}] := N[\{a_1,...,a_M\},b]$ with $\mathfrak{X} = \{a_1,...,a_M\}$ and $b = \mathfrak{Y} \setminus \mathfrak{X}$.

The *strength* of the path $\mathfrak{C}(1),...,\mathfrak{C}(n)$ is
$$\min \{\ \tilde{N}[\mathfrak{C}(i),\mathfrak{C}(i+1)]\ |\ i = 1,...,(n-1)\ \}.$$

In other words: The strength of a path is the strength of its weakest link.

$$P[\mathfrak{A},\mathfrak{B}] := \max \{\ \min \{\ \tilde{N}[\mathfrak{C}(i),\mathfrak{C}(i+1)]\ |\ i = 1,...,(n-1)\ \}\ |\ \mathfrak{C}(1),...,\mathfrak{C}(n) \text{ is a path from set } \mathfrak{A} \text{ to set } \mathfrak{B}\ \}.$$

In other words: $P[\mathfrak{A},\mathfrak{B}]$ is the strength of the strongest path from set $\mathfrak{A} \in A_M$ to set $\mathfrak{B} \in A_M \setminus \{\mathfrak{A}\}$.

(6.2.5) The binary relation O_M on A_M is defined as follows:
$$\mathfrak{A}\mathfrak{B} \in O_M :\Leftrightarrow P[\mathfrak{A},\mathfrak{B}] > P[\mathfrak{B},\mathfrak{A}].$$

(6.2.6) $\mathcal{T}_M := \{\ \mathfrak{A} \in A_M\ |\ \forall\ \mathfrak{B} \in A_M \setminus \{\mathfrak{A}\}: \mathfrak{B}\mathfrak{A} \notin O_M\ \}$ is the *set of winning sets.*

Stage 3:

O_1 is calculated as defined in (6.2.5).

For all $\mathfrak{A},\mathfrak{B} \in \mathcal{T}_M$: Suppose there is a candidate $a \in \mathfrak{A} \setminus \mathfrak{B}$ with $ab \in O_1$ for every candidate $b \in \mathfrak{B} \setminus \mathfrak{A}$, then the set $\mathfrak{A}$ *disqualifies* the set $\mathfrak{B}$.

$\mathcal{S}_M \subseteq \mathcal{T}_M$, the winning sets of the Schulze STV method, is the set of all those sets $\mathfrak{A} \in \mathcal{T}_M$ that are not disqualified by some other set $\mathfrak{B} \in \mathcal{T}_M$.

The fact, that this method is well defined, has been shown by Schulze (2011, section 4.1). The fact, that this method guarantees the election of every Condorcet candidate is a direct consequence of the fact that $N[\{a_1,\dots,a_M\},b] < N/(M+1)$ for every $\{a_1,\dots,a_M\} \in A_M$ and every Condorcet candidate $b \in A$.

6.3. Example

Example (M = 3 seats; C = 5 candidates; N = 630 voters):

60	voters	$a \succ_v b \succ_v c \succ_v d \succ_v e$
45	voters	$a \succ_v c \succ_v e \succ_v b \succ_v d$
30	voters	$a \succ_v d \succ_v b \succ_v e \succ_v c$
15	voters	$a \succ_v e \succ_v d \succ_v c \succ_v b$
12	voters	$b \succ_v a \succ_v e \succ_v d \succ_v c$
48	voters	$b \succ_v c \succ_v d \succ_v e \succ_v a$
39	voters	$b \succ_v d \succ_v a \succ_v c \succ_v e$
21	voters	$b \succ_v e \succ_v c \succ_v a \succ_v d$
27	voters	$c \succ_v a \succ_v d \succ_v b \succ_v e$
9	voters	$c \succ_v b \succ_v a \succ_v e \succ_v d$
51	voters	$c \succ_v d \succ_v e \succ_v a \succ_v b$
33	voters	$c \succ_v e \succ_v b \succ_v d \succ_v a$
42	voters	$d \succ_v a \succ_v c \succ_v e \succ_v b$
18	voters	$d \succ_v b \succ_v e \succ_v c \succ_v a$
6	voters	$d \succ_v c \succ_v b \succ_v a \succ_v e$
54	voters	$d \succ_v e \succ_v a \succ_v b \succ_v c$
57	voters	$e \succ_v a \succ_v b \succ_v c \succ_v d$
36	voters	$e \succ_v b \succ_v d \succ_v a \succ_v c$
24	voters	$e \succ_v c \succ_v a \succ_v d \succ_v b$
3	voters	$e \succ_v d \succ_v c \succ_v b \succ_v a$

In this example, we get:

$N[\{a,b,c\},d] = 169$; $N[\{a,b,c\},e] = 152$; $N[\{a,b,d\},c] = 162$; $N[\{a,b,d\},e] = 159$;

$N[\{a,b,e\},c] = 168$; $N[\{a,b,e\},d] = 153$; $N[\{a,c,d\},b] = 158$; $N[\{a,c,d\},e] = 163$;

$N[\{a,c,e\},b] = 164$; $N[\{a,c,e\},d] = 157$; $N[\{a,d,e\},b] = 167$; $N[\{a,d,e\},c] = 154$;

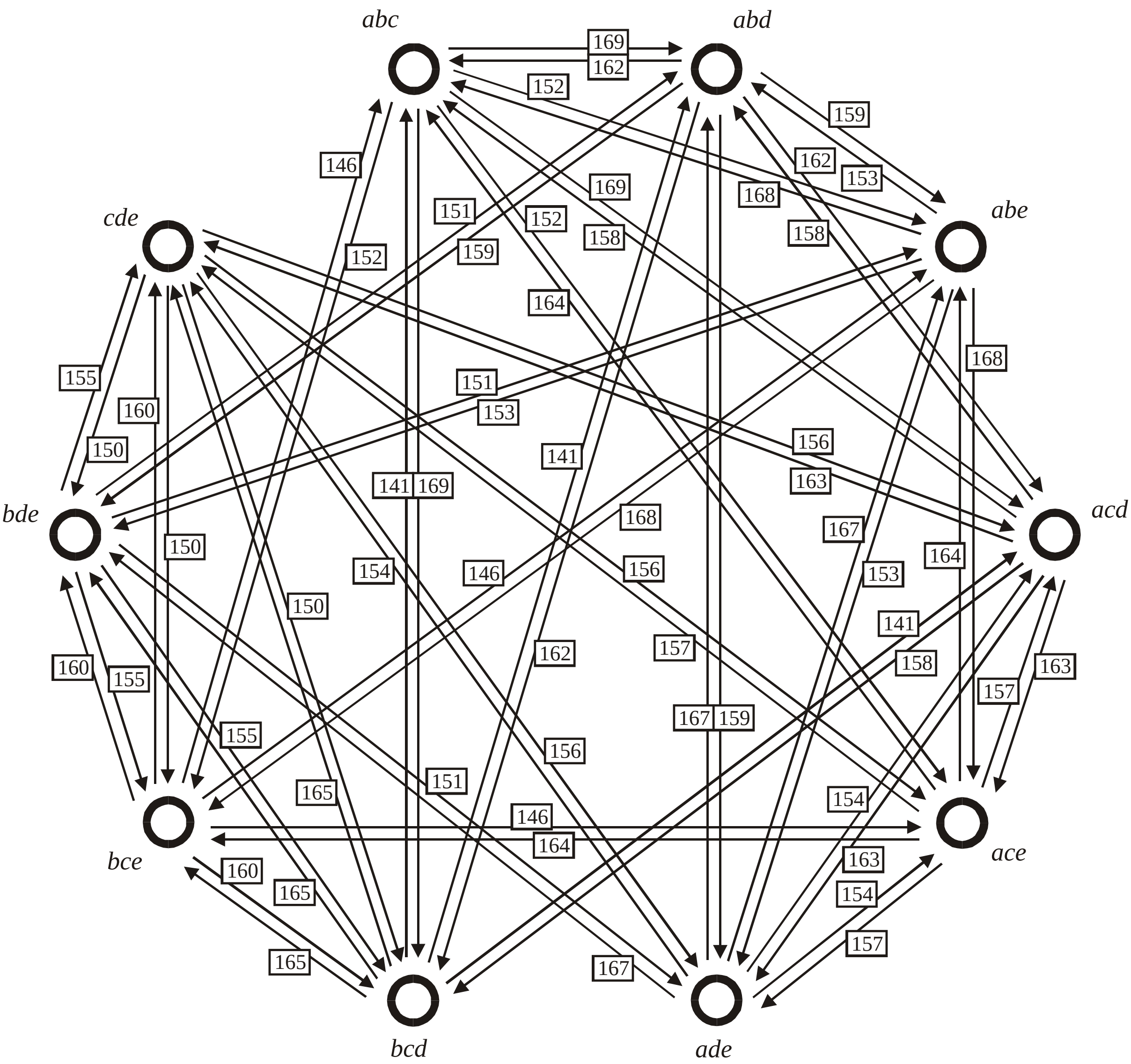
$N[\{b,c,d\},a] = 141$; $N[\{b,c,d\},e] = 165$; $N[\{b,c,e\},a] = 146$; $N[\{b,c,e\},d] = 160$;

$N[\{b,d,e\},a] = 151$; $N[\{b,d,e\},c] = 155$; $N[\{c,d,e\},a] = 156$; $N[\{c,d,e\},b] = 150$.

The pairwise matrix $\tilde{N}$ looks as follows:

	$\tilde{N}[*,\{abc\}]$	$\tilde{N}[*,\{abd\}]$	$\tilde{N}[*,\{abe\}]$	$\tilde{N}[*,\{acd\}]$	$\tilde{N}[*,\{ace\}]$	$\tilde{N}[*,\{ade\}]$	$\tilde{N}[*,\{bcd\}]$	$\tilde{N}[*,\{bce\}]$	$\tilde{N}[*,\{bde\}]$	$\tilde{N}[*,\{cde\}]$
$\tilde{N}[\{abc\},*]$		169	152	169	152		169	152		
$\tilde{N}[\{abd\},*]$	162		159	162		159	162		159	
$\tilde{N}[\{abe\},*]$	168	153			168	153		168	153	
$\tilde{N}[\{acd\},*]$	158	158			163	163	158			163
$\tilde{N}[\{ace\},*]$	164		164	157		157		164		157
$\tilde{N}[\{ade\},*]$		167	167	154	154				167	154
$\tilde{N}[\{bcd\},*]$	141	141		141				165	165	165
$\tilde{N}[\{bce\},*]$	146		146		146		160		160	160
$\tilde{N}[\{bde\},*]$		151	151			151	155	155		155
$\tilde{N}[\{cde\},*]$				156	156	156	150	150	150	

In the graph theoretical interpretation of the proposed election method, each possible way to fill the $M = 3$ seats is represented by a vertex. There is a link from vertex $\mathfrak{A}$ to vertex $\mathfrak{B}$ if and only if set $\mathfrak{A}$ and set $\mathfrak{B}$ differ in exactly one candidate. The strength of the link from vertex $\mathfrak{A}$ to vertex $\mathfrak{B}$ is $\tilde{N}[\mathfrak{A},\mathfrak{B}]$. Therefore, the following graphic represents the discussed example:



The following table lists the strongest paths. “$\mathfrak{A}$,x,$\mathfrak{B}$” means $\tilde{N}[\mathfrak{A},\mathfrak{B}] = x$. The critical pairwise defeats of the strongest paths are underlined. Although there can be more than one strongest path from set $\mathfrak{A} \in A_M$ to set $\mathfrak{B} \in A_M \setminus \{\mathfrak{A}\}$, the strength of the strongest path $P[\mathfrak{A},\mathfrak{B}]$ is well defined.

The strongest paths are:

	... to *abc*	... to *abd*	... to *abe*	... to *acd*	... to *ace*	... to *ade*	... to *bcd*	... to *bce*	... to *bde*	... to *cde*
from *abc* ...	---	*abc*,169, *abd*	*abc*,169, *acd*,163, *ade*,167, *abe*	*abc*,169, *acd*	*abc*,169, *acd*,163, *ace*	*abc*,169, *acd*,163, *ade*	*abc*,169, *bcd*	*abc*,169, *bcd*,165, *bce*	*abc*,169, *bcd*,165, *bde*	*abc*,169, *bcd*,165, *cde*
from *abd* ...	*abd*,162, *abc*	---	*abd*,162, *acd*,163, *ade*,167, *abe*	*abd*,162, *acd*	*abd*,162, *acd*,163, *ace*	*abd*,162, *acd*,163, *ade*	*abd*,162, *bcd*	*abd*,162, *bcd*,165, *bce*	*abd*,162, *bcd*,165, *bde*	*abd*,162, *bcd*,165, *cde*
from *abe* ...	*abe*,168, *abc*	*abe*,168, *abc*,169, *abd*	---	*abe*,168, *abc*,169, *acd*	*abe*,168, *ace*	*abe*,168, *abc*,169, *acd*,163, *ade*	*abe*,168, *abc*,169, *bcd*	*abe*,168, *bce*	*abe*,168, *abc*,169, *bcd*,165, *bde*	*abe*,168, *abc*,169, *bcd*,165, *cde*
from *acd* ...	*acd*,163, *ade*,167, *abe*,168, *abc*	*acd*,163, *ade*,167, *abd*	*acd*,163, *ade*,167, *abe*	---	*acd*,163, *ace*	*acd*,163, *ade*	*acd*,163, *ade*,167, *abe*,168, *abc*,169, *bcd*	*acd*,163, *ade*,167, *abe*,168, *bce*	*acd*,163, *ade*,167, *bde*	*acd*,163, *cde*
from *ace* ...	*ace*,164, *abc*	*ace*,164, *abc*,169, *abd*	*ace*,164, *abe*	*ace*,164, *abc*,169, *acd*	---	*ace*,164, *abc*,169, *acd*,163, *ade*	*ace*,164, *abc*,169, *bcd*	*ace*,164, *bce*	*ace*,164, *abc*,169, *bcd*,165, *bde*	*ace*,164, *abc*,169, *bcd*,165, *cde*
from *ade* ...	*ade*,167, *abe*,168, *abc*	*ade*,167, *abd*	*ade*,167, *abe*	*ade*,167, *abe*,168, *abc*,169, *acd*	*ade*,167, *abe*,168, *ace*	---	*ade*,167, *abe*,168, *abc*,169, *bcd*	*ade*,167, *abe*,168, *bce*	*ade*,167, *bde*	*ade*,167, *abe*,168, *abc*,169, *bcd*,165, *cde*
from *bcd* ...	*bcd*,165, *cde*,156, *ade*,167, *abe*,168, *abc*	*bcd*,165, *cde*,156, *ade*,167, *abd*	*bcd*,165, *cde*,156, *ade*,167, *abe*	*bcd*,165, *cde*,156, *acd*	*bcd*,165, *cde*,156, *ace*	*bcd*,165, *cde*,156, *ade*	---	*bcd*,165, *bce*	*bcd*,165, *bde*	*bcd*,165, *cde*
from *bce* ...	*bce*,160, *cde*,156, *ade*,167, *abe*,168, *abc*	*bce*,160, *cde*,156, *ade*,167, *abd*	*bce*,160, *cde*,156, *ade*,167, *abe*	*bce*,160, *cde*,156, *acd*	*bce*,160, *cde*,156, *ace*	*bce*,160, *cde*,156, *ade*	*bce*,160, *bcd*	---	*bce*,160, *bde*	*bce*,160, *cde*
from *bde* ...	*bde*,155, *cde*,156, *ade*,167, *abe*,168, *abc*	*bde*,155, *cde*,156, *ade*,167, *abd*	*bde*,155, *cde*,156, *ade*,167, *abe*	*bde*,155, *cde*,156, *acd*	*bde*,155, *cde*,156, *ace*	*bde*,155, *cde*,156, *ade*	*bde*,155, *bcd*	*bde*,155, *bce*	---	*bde*,155, *cde*
from *cde* ...	*cde*,156, *ade*,167, *abe*,168, *abc*	*cde*,156, *ade*,167, *abd*	*cde*,156, *ade*,167, *abe*	*cde*,156, *acd*	*cde*,156, *ace*	*cde*,156, *ade*	*cde*,156, *ade*,167, *abe*,168, *abc*,169, *bcd*	*cde*,156, *ade*,167, *abe*,168, *bce*	*cde*,156, *ade*,167, *bde*	---

Therefore, the strengths of the strongest paths are:

	P[*,{abc}]	P[*,{abd}]	P[*,{abe}]	P[*,{acd}]	P[*,{ace}]	P[*,{ade}]	P[*,{bcd}]	P[*,{bce}]	P[*,{bde}]	P[*,{cde}]
P[{abc},*]	---	169	163	169	163	163	169	165	165	165
P[{abd},*]	162	---	162	162	162	162	162	162	162	162
P[{abe},*]	168	168	---	168	168	163	168	168	165	165
P[{acd},*]	163	163	163	---	163	163	163	163	163	163
P[{ace},*]	164	164	164	164	---	163	164	164	164	164
P[{ade},*]	167	167	167	167	167	---	167	167	167	165
P[{bcd},*]	156	156	156	156	156	156	---	165	165	165
P[{bce},*]	156	156	156	156	156	156	160	---	160	160
P[{bde},*]	155	155	155	155	155	155	155	155	---	155
P[{cde},*]	156	156	156	156	156	156	156	156	156	---

$\mathfrak{A} = \{a,d,e\}$ is the unique winning set, since it is the only set with $P[\mathfrak{A},\mathfrak{B}] \geq P[\mathfrak{B},\mathfrak{A}]$ for every other set $\mathfrak{B} \in A_3$.

7. Proportional Rankings

When proportional representation by party lists is being used, then each party has to submit in advance a linear order of its candidates without knowing how many seats it will win. Frequently, the parties are interested that — however many candidates are elected — the elected candidates reflect the strengths of the different party wings in a manner as proportional as possible (Otten, 1998, 2000; Rosenstiel, 1998; Warren, 1999b). We will call a linear order with this property a *proportional ranking*. The two most important suggestions to produce a proportional ranking are the *bottom-up* approach (Rosenstiel, 1998) and the *top-down* approach (Otten, 1998, 2000).

The *bottom-up* approach says that we start with the situation where all C candidates are elected. Then, for $k = C$ to 2, we ask which candidate can be eliminated so that the distortion of the proportionality of the still running candidates is as small as possible; the newly eliminated candidate then gets the k-th place of this party list.

The *top-down* approach says that we use a single-winner election method to fill the first place of this party list. Then, for $k = 2$ to C, we ask which candidate can be added to those candidates who have already got a place so that the distortion of the proportionality is as small as possible; the newly added candidate then gets the k-th place of this party list.

I prefer the top-down approach to the bottom-up approach, because the bottom-up approach starts with the lowest and, therefore, (as the number of candidates is usually significantly larger than the number of seats this party can realistically hope to win) least important places so that slight fluctuations in the filling of the lowest places can have an enormous impact on the filling of the best places. Therefore, in this paper we presume that the top-down approach is being used.

Although we are not aware of any empirical study on strategic voting when producing a party list, we predict that Hylland free riding and vote management are the most serious problems. In this context, *Hylland free riding* means that a voter votes preferably for those of his favorite candidates who are lower in the expected party list. *Vote management* means that a group of candidates asks its supporters to vote preferably for those candidates of this group who are lower in the expected party list. With this strategy the stronger candidates get lower places than they would have got otherwise, while the weaker candidates get better places than they would have got otherwise; therefore, the aim of this strategy is that the stronger candidates get places that are still good enough and that the weaker candidates get places that are just good enough to get elected.

Actually, we predict that Hylland free riding and vote management are significantly more serious problems when producing a party list than under STV methods: The party organization must have an interest that the best places of the party list are filled by those candidates with whom this party wants to advertise in the election campaign. However, when the party expects to win (say) 50 seats and when the voters use Hylland free riding and vote management, then the best places of this party list are rather filled by candidates of that party wing that happens to run the lousiest vote management, because when a candidate gets one of the best places then this means that the number of voters who voted for him was larger than necessary to make him get just one of the best 50 places. Therefore, the party organization must have an interest that the voters vote sincerely (so that the most popular candidates get on the top of the party list) even when it is clear to the individual voter that he wastes a part of his vote when he votes for a candidate who is certain to win one of the first 50 places of the party list even without one’s vote.

Suppose that the top-down approach is being used and that the first k–1 places have already been filled. Suppose $\mathfrak{H}_k$ is the set of all those sets (each of k candidates) that each contain all those k–1 candidates who have already got a place in this party list. Suppose there is a feasible path from set $\mathbb{A} \in \mathfrak{H}_k$ to set $\mathbb{B} \in \mathfrak{H}_k$ and no feasible path from set $\mathbb{B}$ to set $\mathbb{A}$. Then, when we want that the used method to fill the k-th place is not needlessly vulnerable to Hylland free riding and vote management, then $\mathbb{B} \notin \mathcal{S}_k$. In section 7.1, we propose a method that satisfies this criterion.

7.1. Definition of the Schulze Proportional Ranking Method

In this section, we propose new proportional ranking method, where the vulnerability to Hylland free riding and vote management is minimized.

Stage 1:

Proportional completion is used to complete V to W.
Suppose N_W is the number of voters in W.
Suppose $\rho(w) \in \mathbb{R}$ with $\rho(w) > 0$ is the weight of voter $w \in W$.

Stage 2:

For k : = 1 to (C–1) do
{
Suppose $d_1,...,d_{k-1}$ are already elected.

$\mathfrak{H}_k \subset A_k$ is the set of all those sets (each of k candidates) that each contain all elected candidates.

A *path* from set $\mathfrak{X} \in \mathfrak{H}_k$ to set $\mathfrak{Y} \in \mathfrak{H}_k$ is a sequence of sets $\mathfrak{C}(1),...,\mathfrak{C}(n) \in \mathfrak{H}_k$ with the following properties:

1. $\mathfrak{X} \equiv \mathfrak{C}(1)$.
2. $\mathfrak{Y} \equiv \mathfrak{C}(n)$.
3. $2 \leq n < \infty$.
4. For all $i = 1,...,(n-1)$: $\mathfrak{C}(i)$ and $\mathfrak{C}(i+1)$ differ in exactly one candidate. That means: $|\,\mathfrak{C}(i) \cap \mathfrak{C}(i+1)\,| = k - 1$ and $|\,\mathfrak{C}(i) \cup \mathfrak{C}(i+1)\,| = k + 1$.

Suppose $a,b \in A$. Then $N[\{d_1,...,d_{k-1},a\},b] \in \mathbb{R}$ is the largest value such that there is a $t \in \mathbb{R}^{(N_W \times k)}$ such that

(7.1.1) $\forall\, i \in \{1,...,N_W\}\ \forall\, j \in \{1,...,k\}$: $t_{ij} \geq 0$.

(7.1.2) $\forall\, i \in \{1,...,N_W\}$: $\sum_{j=1}^{k} t_{ij} \leq \rho(i)$.

(7.1.3a) $\forall\, i \in \{1,...,N_W\}\ \forall\, j \in \{1,...,(k-1)\}$: $b \succ_i d_j \Rightarrow t_{ij} = 0$.

(7.1.3b) $\forall\, i \in \{1,...,N_W\}$: $b \succ_i a \Rightarrow t_{ik} = 0$.

(7.1.4) $\forall\, j \in \{1,...,k\}$: $\sum_{i=1}^{N_W} t_{ij} \geq N[\{d_1,...,d_{k-1},a\},b]$.

If $\mathfrak{X},\mathfrak{Y} \in A_k$ differ in exactly one candidate, then we define $\tilde{N}[\mathfrak{X},\mathfrak{Y}]$: = $N[\{d_1,...,d_{k-1},a\},b]$ with $\mathfrak{X} = \{d_1,...,d_{k-1},a\}$ and $b = \mathfrak{Y} \setminus \mathfrak{X}$.

The *strength* of the path $\mathfrak{C}(1),...,\mathfrak{C}(n)$ is

$$\min \{ \tilde{N}[\mathfrak{C}(i),\mathfrak{C}(i+1)] \mid i = 1,...,(n-1) \}.$$

In other words: The strength of a path is the strength of its weakest link.

$$P[\mathfrak{A},\mathfrak{B}] := \max \{ \min \{ \tilde{N}[\mathfrak{C}(i),\mathfrak{C}(i+1)] \mid i = 1,...,(n-1) \} \mid \mathfrak{C}(1),...,\mathfrak{C}(n) \text{ is a path from set } \mathfrak{A} \text{ to set } \mathfrak{B} \}.$$

In other words: $P[\mathfrak{A},\mathfrak{B}]$ is the strength of the strongest path from set $\mathfrak{A} \in \boldsymbol{\mathfrak{H}}_k$ to set $\mathfrak{B} \in \boldsymbol{\mathfrak{H}}_k \setminus \{\mathfrak{A}\}$.

(7.1.5) The binary relation O_k on $\boldsymbol{\mathfrak{H}}_k$ is defined as follows:

$$\mathfrak{A}\mathfrak{B} \in O_k :\Leftrightarrow P[\mathfrak{A},\mathfrak{B}] > P[\mathfrak{B},\mathfrak{A}].$$

(7.1.6) $\mathcal{S}_k := \{ \mathfrak{A} \in \boldsymbol{\mathfrak{H}}_k \mid \forall\, \mathfrak{B} \in \boldsymbol{\mathfrak{H}}_k \setminus \{\mathfrak{A}\}: \mathfrak{B}\mathfrak{A} \notin O_k \}$ is the *set of winning sets*.

Candidate $a \in A$ is *eligible* if and only if (1) $a \notin \{d_1,...,d_{k-1}\}$ and (2) $a \in \mathfrak{A}$ for an $\mathfrak{A} \in \mathcal{S}_k$. The k-th place goes to an eligible candidate $a \in A$ with $ba \notin O_1$ for every other eligible candidate b.

}

7.2. Example

Example (C = 5 candidates; N = 504 voters):

6	voters	$a \succ_v d \succ_v b \succ_v c \succ_v e$
72	voters	$a \succ_v d \succ_v e \succ_v b \succ_v c$
12	voters	$a \succ_v d \succ_v e \succ_v c \succ_v b$
6	voters	$a \succ_v e \succ_v b \succ_v d \succ_v c$
30	voters	$b \succ_v d \succ_v c \succ_v e \succ_v a$
48	voters	$b \succ_v e \succ_v a \succ_v d \succ_v c$
24	voters	$b \succ_v e \succ_v d \succ_v c \succ_v a$
168	voters	$c \succ_v a \succ_v e \succ_v b \succ_v d$
108	voters	$d \succ_v b \succ_v e \succ_v c \succ_v a$
30	voters	$e \succ_v a \succ_v b \succ_v d \succ_v c$

Step 1:

$k = 1$.

$\mathfrak{H}_1 = \{ \{a\}, \{b\}, \{c\}, \{d\}, \{e\} \}$.

	$\tilde{N}[*,\{a\}]$	$\tilde{N}[*,\{b\}]$	$\tilde{N}[*,\{c\}]$	$\tilde{N}[*,\{d\}]$	$\tilde{N}[*,\{e\}]$
$\tilde{N}[\{a\},*]$	---	294	174	342	264
$\tilde{N}[\{b\},*]$	210	---	324	306	216
$\tilde{N}[\{c\},*]$	330	180	---	168	204
$\tilde{N}[\{d\},*]$	162	198	336	---	228
$\tilde{N}[\{e\},*]$	240	288	300	276	---

The strongest paths are:

	... to *a*	... to *b*	... to *c*	... to *d*	... to *e*
from *a* ...	---	*a*, 294, *b*	*a*, 342, *d*, 336, *c*	*a*, 342, *d*	*a*, 264, *e*
from *b* ...	*b*, 324, *c*, 330, *a*	---	*b*, 324, *c*	*b*, 324, *c*, 330, *a*, 342, *d*	*b*, 324, *c*, 330, *a*, 264, *e*
from *c* ...	*c*, 330, *a*	*c*, 330, *a*, 294, *b*	---	*c*, 330, *a*, 342, *d*	*c*, 330, *a*, 264, *e*
from *d* ...	*d*, 336, *c*, 330, *a*	*d*, 336, *c*, 330, *a*, 294, *b*	*d*, 336, *c*	---	*d*, 336, *c*, 330, *a*, 264, *e*
from *e* ...	*e*, 300, *c*, 330, *a*	*e*, 300, *c*, 330, *a*, 294, *b*	*e*, 300, *c*	*e*, 300, *c*, 330, *a*, 342, *d*	---

Candidate *e* is the unique winner, since candidate *e* is the unique candidate with $P[e,x] \geq P[x,e]$ for every other candidate *x*. Therefore, candidate *e* gets the first place of the proportional ranking.

Step 2:

$k = 2$.

$\mathfrak{H}_2$ = { {*a*,*e*}, {*b*,*e*}, {*c*,*e*}, {*d*,*e*} }.

	Ñ[*,{*ae*}]	Ñ[*,{*be*}]	Ñ[*,{*ce*}]	Ñ[*,{*de*}]
Ñ[{*ae*},*]	---	147	153	183
Ñ[{*be*},*]	120	---	168	153
Ñ[{*ce*},*]	204	144	---	138
Ñ[{*de*},*]	120	198	168	---

The strongest paths are:

	... to *ae*	... to *be*	... to *ce*	... to *de*
from *ae* ...	---	*ae*, $\underline{183}$, *de*, 198, *be*	*ae*, 183, *de*, $\underline{168}$, *ce*	*ae*, $\underline{183}$, *de*
from *be* ...	*be*, $\underline{168}$, *ce*, 204, *ae*	---	*be*, $\underline{168}$, *ce*	*be*, $\underline{168}$, *ce*, 204, *ae*, 183, *de*
from *ce* ...	*ce*, $\underline{204}$, *ae*	*ce*, 204, *ae*, $\underline{183}$, *de*, 198, *be*	---	*ce*, 204, *ae*, $\underline{183}$, *de*
from *de* ...	*de*, $\underline{168}$, *ce*, 204, *ae*	*de*, $\underline{198}$, *be*	*de*, $\underline{168}$, *ce*	---

Set {*c*,*e*} is the unique winning set, since set {*c*,*e*} is the unique set with $P[\{c,e\},\mathfrak{B}] \geq P[\mathfrak{B},\{c,e\}]$ for every other set $\mathfrak{B} \in \mathfrak{H}_2$. Therefore, candidate *c* gets the second place of the proportional ranking.

Step 3:

$k = 3$.

$\mathfrak{H}_3$ = { {*a*,*c*,*e*}, {*b*,*c*,*e*}, {*c*,*d*,*e*} }.

	$\tilde{N}$[*,{*ace*}]	$\tilde{N}$[*,{*bce*}]	$\tilde{N}$[*,{*cde*}]
$\tilde{N}$[{*ace*},*]	---	98	122
$\tilde{N}$[{*bce*},*]	120	---	102
$\tilde{N}$[{*cde*},*]	120	134	---

The strongest paths are:

	... to *ace*	... to *bce*	... to *cde*
from *ace* ...	---	*ace*, <u>122</u>, *cde*, 134, *bce*	*ace*, <u>122</u>, *cde*
from *bce* ...	*bce*, <u>120</u>, *ace*	---	*bce*, <u>120</u>, *ace*, 122, *cde*
from *cde* ...	*cde*, <u>120</u>, *ace*	*cde*, <u>134</u>, *bce*	---

Set {*a*,*c*,*e*} is the unique winning set, since set {*a*,*c*,*e*} is the unique set with *P*[{*a*,*c*,*e*},$\mathfrak{B}$] ≥ *P*[$\mathfrak{B}$,{*a*,*c*,*e*}] for every other set $\mathfrak{B} \in \mathfrak{H}_3$. Therefore, candidate *a* gets the third place of the proportional ranking.

Step 4:

k = 4.

$\mathfrak{H}_4$ = { {*a*,*b*,*c*,*e*}, {*a*,*c*,*d*,*e*} }.

	$\tilde{N}$[*,{*abce*}]	$\tilde{N}$[*,{*acde*}]
$\tilde{N}$[{*abce*},*]	---	99
$\tilde{N}$[{*acde*},*]	98	---

The strongest paths are:

	... to *abce*	... to *acde*
from *abce* ...	---	*abce*, <u>99</u>, *acde*
from *acde* ...	*acde*, <u>98</u>, *abce*	---

Set {*a*,*b*,*c*,*e*} is the unique winning set, since set {*a*,*b*,*c*,*e*} is the unique set with *P*[{*a*,*b*,*c*,*e*},$\mathfrak{B}$] ≥ *P*[$\mathfrak{B}$,{*a*,*b*,*c*,*e*}] for every other set $\mathfrak{B} \in \mathfrak{H}_4$. Therefore, candidate *b* gets the fourth place of the proportional ranking.

Therefore, the final proportional ranking is $e \succ c \succ a \succ b \succ d$.

8. Database

We applied the Schulze STV method and the Schulze proportional ranking method to the instances of Tideman’s (2000, 2006) database. This database consists of 66 instances. Only in 3 of the 66 instances of Tideman’s database (A10, A13, A34), the winning set of the Schulze STV method differs from the first *M* candidates of the Schulze proportional ranking. In 19 instances, the winning set of the Schulze STV method differs from the winning set of traditional STV methods. Table 8.1 lists all these instances.

	name	*N*	*C*	*M*	Newland-Britton	Meek	Warren	Schulze STV	Schulze proportional ranking
1	A04	43	14	2	*a i*	*i k*	*i k*	*a i*	*i a* ...
2	A05	762	16	7	*a c d e g k m*	*a c d e g k m*	*a c d e g k m*	*a c d e g l m*	*a c m e d g l* ...
3	A06	280	9	5	*c e f h i*	*c e f h i*	*c e f h i*	*b c e h i*	*i h e c b* ...
4	A07	79	17	2	*c i*	*c i*	*c i*	*d i*	*i d* ...
5	A10	83	19	3	*m n p*	*m n p*	*m n p*	*m n p*	(*n a p* ...) or (*n m p* ...)
6	A11	963	10	6	*a c e g h i*	*a c e g h i*	*a c e g h i*	*a c d e h j*	*a c h e j d* ...
7	A13	104	26	2	*k t*	*k t*	*k t*	*k t*	*i t* ...
8	A15	77	21	2	*l r*	*i l*	*i l*	*l r*	*l r* ...
9	A33	9	18	3	[1]	[1]	[1]	*e n o*	*o e n* ...
10	A34	63	14	12	*a b c d e f h j k l m n*	*a b c d e f h j k l m n*	*a b c d e f h j k l m n*	*a b c d e f g h j k m n*	*j b h e n k l m c a d f* ...
11	A35	176	17	5	*a e f n q*	*a e f k n*	*a e f k n*	*a d e f q*	*f e a q d* ...
12	A53	460	10	4	*a b g j*	*a d g j*	*a f g j*	*a f g j*	*j a g f* ...
13	A55	302	10	5	*a d f i j*	*a d e f i*	*a d e f i*	*a e f i j*	*i a f j e* ...
14	A59	694	7	4	*b d f g*	*b d f g*	*b d f g*	*d e f g*	*f d e g* ...
15	A65	198	10	6	*b d e f g j*	*b d e f g j*	*b d e f g j*	*a b e f g j*	*g b f e j a* ...
16	A67	183	14	10	*b c d e f g i j k l*	*b c d e f g h i j k*	*b c d e f g i j k l*	*b c e f g h i j k l*	(*f g k b i e j l c h* ...) or (*g f k b i e j l c h* ...)
17	A71	500	8	7	*a b c d e g h*	*a b c d e g h*	*a b c d e g h*	*a b c d e f g*	*d g c e a b f* ...
18	A74	253	3	2	*a b*	*a b*	*a b*	*a c*	*a c* ...
19	A79	362	8	4	*a e f g*	*a d e g*	*a d e g*	*a c e g*	*g a e c* ...
20	A80	269	7	5	*a b c e f*	*a b c e f*	*a b c e f*	*a b c e g*	*a e c g b* ...
21	A90	366	20	12	*a b c d e f i k l n s t*	*a b c d e f i k l n s t*	*a b c d e f i l n o s t*	*a b c d e f i l n o s t*	*a i t l e c f d s n o b* ...

Table 8.1: Schulze STV method and Schulze proportional ranking method compared to the Newland-Britton (1997) method, the Meek (1969, 1970; Hill, 1987) method, and the Warren (1994) method

[1] In instance A33, 10 candidates receive no first preferences, 7 candidates receive one first preference each, and one candidate receives two first preferences. The winning sets of the Newland-Britton method, the Meek method, and the Warren method depend on which candidates happen to be eliminated by random choice.

9. Conclusions

A *Hylland free rider* is a voter who tries to maximize the influence of his vote by omitting in his individual ranking completely all those candidates who are certain to be elected. *Vote management* is a strategy where a party or a group of independent candidates asks its supporters to vote preferably for those of its candidates who are less assure of election.

In this paper, we demonstrated that today Hylland free riding (section 3.2) and all types of vote management (section 4) are the two most serious problems of proportional representation by the single transferable vote (STV). We introduced a mathematical concept to describe Hylland free riding and vote management (section 6.1) and introduced an STV method (section 6.2) and a method to produce party lists (section 7) where the vulnerability to these strategies is minimized (i.e. methods that are vulnerable to these strategies only in those cases in which otherwise Droop proportionality would have to be violated).

The single-winner case of the proposed methods has already been analyzed by Schulze (2011). Because of the large number of satisfied criteria, both in the single-winner case and in the multi-winner case, we consider the proposed methods to be good methods for public elections.

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