## Election Systems 101 (Rev. C)

Originally presented at SWUUSI 2004
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## Objectives of this workshop:

1. Make attendees aware that there are many different kinds of voting systems, not just a false dilemma between English-style and Italianstyle.
2. Convince attendees that it matters what kind of voting system we use. Witness Papua New Guinea, Duverger's Law.
3. Convince attendees that Instant Runoff Voting (IRV) is better than Plurality (ie. for the US House).
4. Give attendees enough technical knowledge to conduct committee elections using IRV when they get home and to motivate them to do so.
5. Give attendees enough experience with IRV that they would be comfortable doing so.
6. Give attendees the resources and enough understanding to be able to duplicate the workshop when they get home.

## Taxonomy of Election Systems

Single Seat
Multi Seat
Plurality
Majority-Runoff
Approval
(t)
At-Large
Limited Vote
(s)
Cumulative Vote (s)
Closed Party List (p)
Open Party List (p)
Hybrid (p)
Single Transferable Vote (r,p)
Loring's Ensemble Method A (r,p)

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t = two-party system
r = ranked ballot
s = semi-proportional representation
p = proportional representation
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See Glossary for definitions of these systems.

## Glossary (definitions)

Approval Voting: Like SMP, but you can vote for as many candidates as you like (one winner per district).

At-large: There are several overlapping single-winner districts. This is similar to Plurality Voting, except that it is much less likely that there will be many "majority-minority" districts which are likely to elect minorities. This form of "minority vote dilution" is why multi-member districts are illegal for Congressional elections under a 1967 Federal law.

Borda Count: Voters rank candidates as with IRV. Candidates get points in proportion to their ranking. The most points wins (one winner per district).

Bucklin: Voters rank candidates as with IRV. First consider only first-place votes. If no candidate has a majority, include secondplace votes as well, then third-place, etc.

Condorcet: Voters rank candidates as with IRV. Each candidate is compared with each of the others to see who is preferred by a majority. If one candidate wins all of his "pairwise" comparisons, he wins. Otherwise, use a tiebreaker. One winner per district.

Cumulative Voting: PR-lite. There are perhaps three seats per district. Voters get one plurality-style vote for each seat, but may lump their votes together on one candidate if they wish. Results are erratic.

Hybrid: Germany uses a Mixed-Member hybrid system in which voters vote in single-member plurality districts, but some additional seats are assigned from party lists in order to make the resulting party representation proportional to their vote totals.

Instant Runoff Voting (IRV): Voters rank the candidates in order of preference (1,2,3...). Ballots go to the most preferred candidate. Whoever gets the fewest ballots is eliminated, and his ballots get redistributed. Repeat until there is one winner.

Limited Voting: More PR-lite. You can vote for several candidates (no lumping several votes on one candidate), but there are more seats to be won than you get votes. Results are erratic.

Loring Ensemble Rule A (LERa): Like STV, but the Condorcet winner is exempted from the elimination process. This assures that the resulting legislature will have a substantial number of centrists.

Majority-runoff: Vote for one candidate only, one winner per district. If no one gets a majority, hold a separate runoff election between the top two candidates.

Nanson: Like the Borda Count, but instead of "the most points wins," the candidate with the fewest is eliminated, and the ballots recounted as if the eliminated candidates had never existed. Repeat until there
is one winner.

Party List PR: Voters basically vote for parties rather than candidates, and parties get seats proportionate to their vote totals. The entire legislature may be a single multi-seat district. Under "closed list" PR, the parties choose which of their candidates get seated first from a predetermined list. Under "open list" PR, each voter votes for a specific candidate, and the party's seats are assigned in order of popularity.

Proportional Representation (PR): Multi-seat districts are used. Seats in the legislature are assigned to representatives of various groups of voters in proportion to the size of those groups.

Single Member Plurality (SMP): You know this one. Vote for one candidate only, and whoever gets the most votes wins. No runoffs. One winner per district. Also known as First Past The Post (FPTP).

Single Transferable Vote (STV): A PR system with small multi-seat districts, with ballots that look like IRV ballots and are processed similarly. There are multiple winners because it takes fewer than 51\% of the votes to claim a seat. Excess votes for winners are redistributed as well as votes for candidates who are eliminated.

Smith Set: The set of candidates who participate in a circular tie using the Condorcet method. In small committee elections, where there are likely to be lots of ties, it may be necessary to distinguish between the Smith set and the Schwartz set. Everyone in the Schwartz set has beaten everyone who is not. No one in the Smith set was defeated by anyone who is not.

## Mock Election Using Plurality.

Whichever candidate gets the most votes wins, even if most voters voted against it. This is the system we use to elect the US Congress. It is usually referred to either as Single Member Plurality (SMP) or as First Past The Post (FPTP).

Pick one of the following candidates for Caterer of the Canvass Dinner:

Chinese
Italian
Mexican
Greek

## Mock Election Using Majority-Runoff.

If one candidate gets a majority, it wins. Otherwise, hold a runoff election between the top two vote getters. This is the system we use to elect the Mayor of Houston.

Pick one of the following candidates for Caterer of the Canvass Dinner:

Mexican
Greek

## Motivation for electoral reform:

Two parties are not enough.
"Voting systems are to a democracy what the 'operating system' is to a computer...."
-- Steven Hill, Fixing Elections, p. ix.
"If partisan conflict is multidimensional, a two-party system must be regarded as an electoral straitjacket that can hardly be regarded as democratically superior to a multiparty system reflecting all the major issue dimensions."
-- Arend Lijphart
"The two major political parties, Republicans and Democrats, have formed what amounts to an implicit cartel to reduce competition from third parties, and on the whole, it is very effective."
-- McKenzie and Tullock, p. 169.
"The notion of a "left" and a "right" ... originated in the seating arrangement of the French National Assembly during their revolution.... Within a year it was invalidated by events."
-- Jerry Pournelle, http://www.baen.com/chapters/axes.htm
"People's attitudes ... tend to be strongly correlated, although logically there should be little connection between a person's attitude to abortion and his attitude to multi-culturalism."
-- John McCarthy,
http://www-formal.stanford.edu/jmc/progress/ideology.html



I don't like the two-party system because political parties are inherently oligarchic; the two-party system allows special interests to dictate the terms of political debate, and doesn't give voters enough meaningful choices (ie. drug policy reform).

Defenders of the two-party system often present a false dilemma between the two-party system and Proportional Representation (PR), which has a partially deserved reputation for producing "unstable" coalition governments. Proponents of $P R$ often do the same thing, making indiscriminate attacks on all single-member district election systems, including ones that they advocate in situations where PR is not possible (ie. elections for executive offices such as mayor).

The two-party system is also notorious for inviting gerrymandering.
Other arguments for or against the two-party system are more controversial:

To what extent does the two-party system promote "the politics of mobilization" vs. "the politics of persuasion" relative to other systems? The politics of mobilization means appealing to extremists, trying to increase turnout of one's base. The politics of persuasion means appealing to the center, trying to cut a deal with one's opponents' supporters. The amount of violence associated with elections in Papua New Guinea (PNG) increased markedly after PNG abandoned Australian-style Instant Runoff Voting in favor of USstyle Plurality Voting. (See the PNG case study in the I-IDEA Handbook: running spoilers and intimidation vs. appealing for second preferences.)

To what extent can front-runner status in primary elections be bought, making primaries unnecessarily sensitive to money?
"Where once it was useful to be the second choice of $90 \%$ of all delegates, today first choices--even of as few as 30\%--are far preferable."

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        -- Polsby and Wildavsky (p. 115) regarding Presidential
        primaries
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How well does the two-party system provide policy stability, tracking popular opinion (the median voter) as opposed to bouncing around between extremist factions?

Is the two-party system (and single-member district elections in general) bad for minorities (minority vote dilution), or does it simply represent them differently than PR systems do (implicit vs. explicit bargaining)? (PR is often advertised as the alternative to racial gerrymandering, and this appears to be the primary motivation for electoral reform on the part of many activists.)

Reformers often blame the two-party system for low voter turnout, alienation, and money being too important.

## Duverger's Law

Maurice Duverger wrote that the tendency for Single-Member Plurality (SMP) voting to produce two-party elections is so strong that it comes as close as anything does in the field of sociology to being a natural law. With Plurality, there are no runoffs, so your second choice doesn't matter. You don't even get to express what your second choice is. The "mechanical" effect of underrepresenting minor parties in the legislature is reinforced by the "psychological" effect that people don't like giving their support to a party that consistently gets screwed. Minor parties tend to be shunned as "spoilers;" support for them takes support away from similar major parties. (I discuss this in more detail in Appendix A.)

In Canada's parliamentary system, there are no presidential "coattails," so while there is a two-party system in each province, they are not the same two parties throughout the whole country.

Another quasi-natural law is that political parties are oligarchic.
A third observation is that political parties contain mixes of people with different attitudes: "fight to win" vs. "fight to feel good about losing."

Does the flavor of wine stored in a single large bottle change if it is decanted into several small bottles? Duverger argued that wine bottles are a bad analogy for political parties. The ability of small parties to win seats in the legislature changes the way in which political movements evolve.

## Mock Election Using Approval Voting.

Whichever candidate gets the most votes wins, even if most voters voted against it, but voters can vote for as many candidates as they want.

Pick any number of the following candidates for Caterer of the Canvass Dinner:

| Chinese | Italian | Mexican | Greek |
| :--- | :--- | :--- | :--- |
| Barbeque | French | German | Salad |

## Instant Runoff Voting (IRV).

aka Alternative Vote, Hare Elimination, Australian Ballot
Voters rank the candidates in order of preference. Ballots are assigned to the top-ranked candidate and counted. The candidates with the fewest votes are successively eliminated and each of their ballots is transfered to the remaining candidate who is ranked next on that ballot until one candidate has a majority.

Conceptually, the candidates line up at the front of the room, and each voter lines up behind his or her favorite candidate. The least supported candidate is eliminated, and those voters each move to their next favorite candidates. Repeat until there is a majority winner.

So far, we have not considered tiebreakers. Obviously, in a small committee-sized election, there is the possibility of a tie, but until now, the candidates could flip a coin after the votes are processed. With an elimination system such as IRV, there is the possibility of a tie at each stage in the elimination process, and if the candidates are not all physically present when the votes are counted, the tiebreaking process needs to be formal and verifiable. Often a simple tiebreaker will itself produce a tie, and so multiple tiebreakers may be needed. One convenient tiebreaker is to give preferrence to the candidate who had the most first place votes, with second place, third place, etc. votes used as additional tiebreakers (a modification of Bucklin's voting method). Note that there is a psychological advantage to being listed first on the ballot, so the question of which of two candidates will have an unfair advantage over the other will already have been addressed in some sense. In an emergency, one could break a tie by giving the advantage to the candidate listed first (or last) on the ballot. However, no matter what tiebreaking methods you use, they must be specified in advance in order to avoid accusations of favoritism. If necessary, I will use the Bucklin scheme first, and in an emergency pick the candidate listed last.

## Mock Election Using Instant Runoff Voting

Rank the following candidates for Caterer of the Canvass Dinner in your order of preference:
Chinese Italian Mexican Greek

1st choice:
2nd choice:
3rd choice: $\qquad$
4th choice:

## Case Study of Papua New Guinea (PNG).

The clearest example I know of where an election system made a difference is Papua New Guinea. There is a case study on pp. 40-42 of The International IDEA Handbook of Electoral System Design (Institute for Democracy and Electoral Assistance, http://www.int-idea.se ) by Ben Reilly, "Papua New Guinea: Electoral Incentives for Inter-Ethnic Accomodation." As an Australian territory and when PNG first became independent, they used IRV (aka the Alternative Vote, AV), but they later switched to Plurality (what we use, Single-Member Plurality, SMP, aka First Past the Post, FPTP) because it was simpler. The PNG society is clan-based, with clans that are much smaller than the electoral districts, so even though the parties were clan-oriented, under IRV (AV) they had to compete with one another for the second or third preferences on other clan's ballot papers. This forced them to act in an accomodating manner to other clans. After the system was changed to Plurality (FPTP), the winner was whoever had the largest (minority) block, regardless of the feelings of the other clans. It became possible to win with a very narrow base of support--as little as 6.3\%. The largest clans no longer courted other clans for second preferences, but tried to discourage their rivals from voting (electoral violence increased), or tried to encourage spoilers in the other clans.

Reilly writes, "The Papua New Guinea case illustrates just how dependent much of the accepted wisdom regarding electoral systems is on the structure of the society concerned." This is a common thread throughout the IDEA book: no one electoral system is promoted as generally the best, but one system may very well be preferred over another in the context of a specific society. Some critics of electoral reform claim that there is no reason to prefer one system over another except for partisan reasons. In my opinion, the PNG case firmly contradicts this claim.

Another IDEA case study involved South Africa's party list Proportional Representation system (list PR), and how the African National Congress yielded the partisan advantages they would have gained by insisting on a different electoral system in order to produce a greater sense of legitimacy and acceptance for the new governments. The book also notes (p. 125) that electoral reform is not a panacea, but "while most of the changes that can be achieved by tailoring electoral systems are necessarily at the margins, it is often these marginal impacts that make the difference between democracy being consolidated or being undermined." Although these marginal impacts are most important in a society whose grip on democracy is tenuous, I find electoral reform activism compeling because this is one of the few areas in political science where the problems actually have technical solutions.

A recent email from the Center for Voting and Democracy reports that the Utah Republican Party is using IRV, and that in Utah it has also resulted in "kindlier, gentler campaigning."

## The "notorious" Borda Count

"My scheme is intended only for honest men." -- Jean Charles de Borda

Voters rank the candidates in order of preference. If there are $N$ candidates, the 1st choice gets $\mathrm{N}-1$ points, the 2 nd gets $\mathrm{N}-2$, etc. Whichever candidate gets the most points wins. (Philosophically, these are ordinal numbers, but Borda interprets them as cardinal numbers.)

The finance committee, Larry, Moe, and Curly, coming back from the district conference, took a different exit and ended up at Goldberg's Real Italian Pizza restaurant. Larry and Moe are vegetarians, but Curly wants to order a pepperoni pizza. They decide to use the Borda Count to choose a pizza. Larry points out that with only two choices, the Borda Count is identical to Plurality Voting.

Voter Preferences

| choice | (points) | Larry | Moe | Curly |
| :---: | :---: | :---: | :---: | :---: |
| 1st | 1 | Veggie | Veggie | Pepperoni |
| 2nd | 0 | Pepperoni | Pepperoni | Veggie |

Borda Count

| Veggie | $1+1+0=2$ | $-->$ | winner |
| :--- | :--- | :--- | :--- | :--- |
| Pepperoni | $0+0+1=1$ |  |  |

Curly convinces Larry and Moe that they should look at the menu before voting, and the list of candidates that eventually emerges is Veggie, Pepperoni, Pepperoni and Canadian Bacon, and Pepperoni and Ham. Larry and Moe want to have to pick the minimum number of pieces of meat off the pizza, so they choose Pepperoni as their second choice. Curly doesn't much care for Canadian bacon or ham either, but will put up with them in order to get pepperoni. The ballots are thus marked.

## Voter Preferences

| choice | (points) | Larry | Moe | Curly |
| :---: | :---: | :---: | :---: | :---: |
| 1st | 3 | Veggie | Veggie | Pepperoni |
| 2nd | 2 | Pepperoni | Pepperoni | Pepperoni+CB |
| 3rd | 1 | Pepperoni+CB | Pepperoni+CB | Pepperoni+Ham |
| 4 th | 0 | Pepperoni+Ham | Pepperoni+Ham | Veggie |


| Veggie | $3+3+0=6$ |  |  |
| :--- | :--- | :--- | :--- |
| Pepperoni | $2+2+3=7$ | --> | winner |
| Pepperoni+CB | $1+1+2=4$ |  |  |
| Pepperoni+Ham | $0+0+1=1$ |  |  |

Moe whispers to Larry, "Next time we vote on a pizza, I'm going to lie about my preferences in order to put Curly's favorite at the bottom of my list." Larry looks thoughtful for a moment and then says, "You know, I was going to run for City Council next month, and I just realized that the voting will be done using the Borda Count. Why don't all three of us run? My cousin is in the real estate business, and he'd be happy to hire some college students to circulate our ballot access petitions."

Borda has a serious problem with insincere voting. Results vary between being similar to Plurality Voting to being largely random. If many voters vote strategically, and rank a seemingly non-serious "Bozo" candidate above a candidate that they view as a serious rival, the Bozo could easily win. More importantly, Borda is fatally sensitive to "clone" candidates. Whereas a spoiler problem deters spoilers from running, a voting system with a clone problem encourages clones to run. The outcome of a Borda Count election is thus likely to be determined largely by ballot access issues.

This is a shame, because for many people, it is the most intuitive of all of the voting systems that use ranked ballots. Mention to an engineer the idea of ranking candidates in order of preference, and he or she is likely to reinvent the Borda Count right on the spot. So before you can talk about better, but less intuitive voting systems, you will first need to explain why serious election reform advocates don't recommend Borda.

Condorcet's Paradox
(rock-paper-scissors, aka "cycling")

Which restaurant should the music committee stop at on the way home from the District Conference?

|  | Peter | Paul | Mary |
| :--- | :---: | :---: | :---: |
| 1st choice | Chinese | Italian | Mexican |
| 2nd choice | Italian | Mexican | Chinese |
| 3rd choice | Mexican | Chinese | Italian |

The arrows in this "tournament graph" are drawn from the more preferred to the less preferred candidate.

Figure 3
A Circular Tie (Tournament Graph)


Each member has "transitive" preferences (they can be ranked in order), but the committee as a whole does not. No matter which restaurant is proposed, a $2 / 3$ majority prefers one of the alternatives (and can agree on which one).

Cycling does not occur if all of the relevant political positions lie on a single axis. It also does not occur in a three-candidate race if two of the candidates are much closer to one another than either is to the third (See the March 2004 Scientific American).

## Mock Election Using Condorcet's Method.

aka Pairwise-Runoff, True Majority Rule

We are looking for one candidate who would defeat any other candidate in a two-candidate election. If no such candidate exists, use Tideman's "Ranked Pairs" circular tiebreaker method.

Rank the following candidates for Caterer of the Canvass Dinner in your order of preference:
Chinese Italian Mexican Greek

1st choice:
2nd choice:
3rd choice:
4th choice:

If there is a circular tie, the set of candidates who participate in this tie is called the Smith set. Many types of tiebreaker have been proposed. Some tiebreakers force the winner to be a member of the Smith set, but others don't. Nanson's Method (aka Borda Elimination), Schulze's "beatpath" method, and Tideman's "ranked pairs" method do. Identifying the Smith set is a minor hassle, but none of these methods require that it be done explicitly. For purposes of illustration, I will ignore the possibility that ballots may not be completely filled in. Incomplete ballots have to be accounted for when using Nanson's Method (see my Example 3). Using Schulze and Tideman, the strength of a victory is supposed to be determined by the size of the majority, not the size of the vote differential (see Mike Ossipoff's web site for details).

Borda Elimination works by calculating the Borda count, eliminating the candidate with the fewest points, and recalculating as if that candidate had never been on the ballot in the first place. Repeat until only one candidate remains. (Some references call this Baldwin's rather than Nanson's method.)

Schulze's Beatpath Method (verbatim from Mike Ossipoff's site):

1. X has a beatpath to Y if either $X$ beats $Y$, or $X$ beats someone who has a beatpath to Y. A sequence of defeats that makes it possible to correctly say that $X$ has a beatpath to $Y$ is called a beatpath from $X$ to Y.
2. The strength of a beatpath is measured by the strength of its weakest defeat.
3. If the strongest beatpath from $X$ to $Y$ is stronger than the strongest beatpath from $Y$ to $X$, then $X$ has a beatpath win against $Y$.
4. A candidate wins if no one has a beatpath win against him.

Tideman's "Ranked Pairs" Method lists the pairwise defeats in order from most decisive to least decisive, "locking" any of these defeats that do not form cycles with the previously locked defeats, and discarding defeats that do. The winner is the candidate with no locked defeats.

As noted above regarding Instant Runoff Voting, in committee-sized elections, one often needs one or more conventional tiebreakers during the processing of the ballots, quite apart from the possible need for a circular tiebreaker. Again, I will first try to resolve ties using the modified Bucklin's method, then resort to favoring the candidate listed last on the ballot.

## Arrow's Impossibility Theorem

Kenneth Arrow (via William Vickrey) suggested that a good "Social Welfare Function" (ie. a group decision-making process) should satisfy the following five postulates:

1. Do not overrule unanimous decisions.
2. Is not a dictatorship.
3. Is transitive (no circular ties, aka "rock-paper-scissors").
4. There are no restrictions on the range of alternatives that can be compared.
5. How two alternatives compare to each other does not depend on how they compare to a third alternative.

No social welfare function satisfies these five postulates. In other words, no matter what election system you use, sooner or later it will produce an outcome that is at least mildly perverse. See Mueller, p. 386. See also the March, 2004 issue of Scientific American. We will look at some of these perverse scenarios in the section on "interesting" example sets of ranked ballots.

## Implicit vs. Explicit Bargaining

Many discussions of electoral reform fail to make clear distinctions between arguments for Proportional Representation and arguments for other alternatives to Plurality voting that still involve single seat districts. The best explanation of the fundamental differences between these two basic classes of election systems that $I$ have seen was in The Calculus of Consent, by James M. Buchanan and Gordon Tullock. Here is my version of it.

Consider a society made up of three factions: A, B, and C. Each of the three factions is geographically dispersed, having more or less the same proportions in all parts of the country. Differences within any one faction are relatively minor.

35\% are in faction A. These people are gay, and willing to pay $\$ 1000 / y e a r$ to avoid being persecuted. They also favor a ban on handguns, and are willing to spend $\$ 100 / y e a r$ to have a ban.

Another $40 \%$ of the population are in faction B. These are gun owners who are willing to spend $\$ 500 / y e a r ~ t o ~ a v o i d ~ a ~ h a n d g u n ~ b a n, ~ b u t ~$ they are also homophobic, and willing to spend $\$ 50 / y e a r$ in order to persecute gays.

The remaining 25\% are in faction C. These people both favor a handgun ban and are homophobic, willing to spend \$100/year for a handgun ban and $\$ 50 / y e a r ~ t o ~ p e r s e c u t e ~ g a y s . ~$

If this society uses initiative and referenda, both of these issues can be raised independently. A proposal to ban handguns would pass, $60 \%$ to $40 \%$, and so would a proposal to persecute gays, $65 \%$ to $35 \%$. But $75 \%$ of the population would be unhappy with this result.

It turns out that we have a Condorcet paradox here. If the choice were limited to passing just one of the referenda (either one) or passing neither, either one of the referenda would pass by at least $60 \%$ to $40 \%$. If the choice were limited to passing both referenda or just a specific one of them, both would pass, again by at least $60 \%$ to $40 \%$. But if the choice were limited to both referenda passing or neither of them, neither would be selected, $75 \%$ to $25 \%$.

Can we say objectively that the passage of either of these referenda is bad? If we don't have too much heartburn about making interpersonal utility comparisons, and if we can get away with assuming that a dollar is worth roughly the same to an average member of any of the three groups, then we can use a criterion called "Marshall efficiency" (after economist Alfred Marshall) and just sum up the dollar values that each person would be willing to spend to get his or her way.

For every 100 average people in the society, the handgun ban is worth

$$
\$ 100 * 35-\$ 500 * 40+\$ 100 * 25=-\$ 14,000 .
$$

Similarly, persecuting gays is worth

$$
-\$ 1000 * 35+\$ 50 * 40+\$ 50 * 25=-\$ 31,750 .
$$

These two calculations are independent. In real life, we almost never have enough information to make this sort of calculation. We can reasonably ask voters to rank some of their preferences in order (ordinal numbers), but it's hard to get people to assign realistic weights to their preferences (cardinal numbers).

Now let's consider what would happen if this society were to use representative democracy rather than direct democracy, with a legislature elected in single-member districts. Let's also suppose that they use Plurality voting, and thus have a two-party system. In one district, suppose candidate Smith runs on a platform of banning handguns and persecuting gays. His only opponent, candidate Jones, runs on a platform of not banning handguns and not persecuting gays. Forced to choose between these two, both factions A and B prefer Jones, giving him an $75 \%$ to $25 \%$ victory. If this society is lucky enough to have similar pairs of candidates running in all districts, with similar results, the legislature will be homogeneous, and will refuse, with little discussion, to pass either a handgun ban or gay persecuting legislation. Factions A and B have agreed to an implicit bargain, proposed by candidate Jones in his platform, to leave each other alone. The deal was struck on election day. This is "implicit bargaining."

On the other hand, if this society were to use a Party List Proportional Representation system, and these two issues are the most important ones under discussion, we would be likely to see three political parties: A, B, and C. Party A supports a handgun ban, Party B supports persecution of gays, and Party C supports both. There is little need for policy discussion within any of these parties before the election, but after the election, a heterogeneous legislature emerges. The legislature is likely to witness loud debates and intense logrolling efforts. Depending on how the logrolling works, the representatives are likely to reach the same compromise as in the previous case, passing neither the handgun ban nor persecution of gays. This is "explicit bargaining."

Given that we have a Condorcet paradox, and having claimed that both of the referenda would have passed under direct democracy, why do I suggest that neither would be passed into law under representative democracy? The reason is that under representative democracy, whether the decisions are made by candidates choosing their platforms or representatives engaged in logrolling, the number of people directly involved in decision making is relatively small. With small numbers of direct participants, I expect bargaining of some form to actually take place. (See Mansur Olson, The Logic of Collective Action) Other issues will come up, and some ways of making side payments are likely to be found to lure weakly opposed voters or representatives to switch sides. This could involve preferential tax treatment, strangely located military bases, or more prompt road repairs. But direct bargaining that is impractical with large numbers of voters is practical with small numbers of political figures, and the Marshall efficient outcome has an advantage, at least in theory, over the other outcomes in terms of its proponents being motivated to offer side payments.

Is this realistic? How well explicit bargaining will work in practice may depend on how effective party leaders are at forcing their members to vote as a block (party discipline), who controls the legislative process (agenda setting), how much information the politicians have, and whether ways of making side payments can be found that are not too wasteful or too heavily stigmatized. The socially attractive (Marshall efficient) outcomes have an advantage, but not necessarily a decisive one. How well implicit bargaining works may depend on how many parties there are and on the election system used (as well as party discipline and well-informed politicians).

Block voting violates some of the assumptions that went into the analysis of implicit and explicit bargaining in The Calculus of Consent. With block voting, it isn't possible to use side payments to lure just a few weakly opposed voters into switching sides. Thus, relatively desirable (Marshall efficient) policies no longer have a strong bargaining advantage over competing policies.

Some single-seat election systems are very good at picking Condorcet winners, candidates who would have beaten any of their rivals in one-on-one elections, if such candidates exist. However, in the case of a Condorcet paradox, no such candidate exists, and we need a "circular tiebreaker." Later on, I will present three such tiebreakers, but none of them produce the Marshall efficient outcome in this example. Nanson's Method selects the gun ban, and the other two methods (Schulze and Tideman) select homophobia.

Much of the support for Proportional Representation (PR) in the US is due to the belief that it provides better representation for racial minorities. For example, if a racial minority is uniformly distributed throughout many single-seat election districts in a society that is polarized along racial lines, there will be few if any "majority minority" districts, and consequently few members of that minority in the legislature. This effect is called "minority vote dilution." This argument for PR assumes that explicit representation is better than implicit representation. But is PR really better? Theoretically, implicit and explicit bargaining should both work just fine in terms of representing the interests of political minorities. In practice, in a polarized society, neither system is likely to protect minorities from the "tyranny of the majority." Under implicit bargaining, a platform designed to attract a minority of $20 \%$ is likely to alienate a large fraction of a hostile $80 \%$, and the minority's interests are rejected on election day. Under explicit representation, the minority get shafted by the same percentages in the legislature, after the election, but they still get shafted. It may be that explicit bargaining is better for minorities, for example because the process is more public, but these advantages don't appear to me to be overwhelming.

In practice, populations are not uniformly distributed, and factions are internally divided, so there is always a mixture of implicit and explicit bargaining. Also, remember that some important issues arise unexpectedly after the election, such as how to respond to the $9 / 11$ attacks.

## Discussion of Proportional Representation.

"Nature always sides with the hidden flaw."
-- Murphy's law corollary
We have already discussed the differences between legislatures elected using Proportional Representation (PR) and those elected using single-winner election systems in terms of explicit vs. implicit bargaining. Most of the criticisms of PR are related to the fact that more bargaining takes place in PR legislatures than in Plurality legislatures, where more of the compromises are implicit in the politicians' platforms. Outcomes under PR are thus more sensitive to quirks in the way legislatures work.

For example, in Weimar Germany, under PR, extremist parties would agree to sack the Chancellor, then be unable to agree on a replacement. Modern Germany has a rule that you have to agree on a replacement first.

One set of quirks is associated with the legislature's agenda setter (ie. Speaker of the House or Prime Minister). (See Dennis Mueller's discussion of agenda manipulation.) Israel has a national election for their Prime Minister (although the legislature still has to form a government, which seems to me to defeat the purpose).

Gridlock is also more likely in some kinds of legislatures than in others (see Cox). A certain amount of susceptibility to gridlock is probably necessary to prevent the tyranny of the majority, but it's easy to have too much. The US has a bicameral, presidential "separation of powers" system that tends to produce gridlock. PR legislatures are more likely to suffer from gridlock than singlewinner legislatures are, because no one party is likely to have a majority. Gridlock is especially likely with "disciplined" political parties, which consistently vote in blocks. US parties tend to be undisciplined because our primary elections tend to undermine the authority of party leaders. Parliamentary systems can limit gridlock by calling new elections when it gets bad.

How well do the coalition governments produced by PR elections actually track the median voter? Huber and Powell argue that they do at least as well as most plurality systems, with centrist parties holding the balance of power in the coalitions.

However, coalition governments are sensitive to the logic of coalition forming (see John Ferejohn). There may not be a large enough number of centrist legislators to matter much in a coalition. A coalition may thus be overly influenced by extremists, or be unable to "form a government," or once formed, the coalition may be unstable. Or, the same coalition could form repeatedly despite large shifts in public opinion, making a coalition "too stable." Some of these criticisms may not appear to be relevant to the US system, where the president chooses his cabinet and the dates for the elections are fixed in the Constitution, but they may still matter depending on the way the legislatures' agenda setters are chosen.
Figure 4
Difficult Coalition Forming Scenario
Fringe Party 1
$\mathbf{x}$
Labor
Fringe Party 2

Robert Loring has proposed his "Loring Ensemble Rule A" as a way to guarantee a substantial number of centrists in a PR legislature. The German "Mixed Member" system appears to have a similar effect.

Figure 5
Improved Coalition Forming Scenario

| Fringe Party 1 |  |
| :---: | :---: |
| $\mathbf{x}$ |  |
| Labor | Centrist Conservative |
| x |  |
| Fringe Party 2 |  |

John Ferejohn has concerns about transparency and accountability in coalition governments. South Africa chose PR so that their governments would be widely regarded as legitimate. But are we more worried about governments lacking legitimacy for lack of minority legislators, or lacking legitimacy for being unresponsive due to coalitions being too stable?

What are the psychological implications of fringe parties being able to win seats in the legislature? Most Party List systems seem to have multi-seat thresholds (1\% for Israel, 5\% for Germany) for how much support a party has to have in order to win any seats.

While I like PR in principle, whether or not $I$ buy the arguments for it will depend on the details of the particular government it's being proposed for, and what reforms it's bundled with.

Note that there is a 1967 Federal law that bans multiseat districts for Congressional elections. The purpose was largely to prohibit atlarge districts, which were being used for minority vote dilution. A Constitutional amendment is not needed in order to use PR or alternative single-winner election systems in US House elections.

You may find a more thorough discussion of Proportional Representation in an advocacy piece for Instant Runoff Voting I have posted at http://www.ghg.net/redflame/irv.htm .

## Example set 1:

Compromise candidates and runoff elimination order.
Suppose a society is divided into four "ideological tribes" in the following way. Each tribe has its own favored candidate. The two largest tribes are North and South. The North tribe's candidate is slightly closer to the center than the South tribe's candidate. Also, the East tribe's candidate is slightly closer to the center than the West tribe's. There is no tribe of loyal centrist voters, but one independent candidate has chosen to take a centrist position:

Figure 6
Hypothetical "Ideological Tribes" and their Support

North
31
West Center East

20
0
9

40
South

If the voters' secondary preferences are determined by the distances shown on this figure, we have the following preference schedule:

Voters' Preference Schedules:

| Choice | Number of Voters |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
|  | $\mathbf{3 1}$ | $\mathbf{4 0}$ | $\mathbf{9}$ | $\mathbf{2 0}$ |
| 1st | North | South | East | West |
| 2nd | Center | Center | Center | Center |
| 3 rd | East | East | North | North |
| 4 th | West | West | South | South |
| 5 th | South | North | West | East |

The centrist is no one's first choice, but everyone's second choice. Now suppose that an election is held using either Condorcet or Instant Runoff Voting, and that everyone votes his sincere preferences.

First consider what will happen if these votes are counted using

Condorcet

| Center | beats | North | $\mathbf{6 9 - 3 1}$ |
| :--- | :--- | :--- | :---: |
| Center | beats | South | $\mathbf{6 0 - 4 0}$ |
| Center | beats | East | $\mathbf{9 1 - 9}$ |
| Center | beats | West | $\mathbf{8 0 - 2 0}$ |
| North | beats | South | $60-40$ |
| North | beats | East | $51-49$ |
| West | beats | North | $60-40$ |
| South | beats | East | $60-40$ |
| West | beats | South | $51-49$ |
| East | beats | West | $80-20$ |

Center is the Condorcet winner, undefeated in pairwise comparisons. Now consider what will happen if Instant Runoff is used instead:

## Instant Runoff Voting

| Round | North | South | East | West | Center |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 31 | 40 | 9 | 20 | 0 |
| 2 | 31 | 40 | 9 | 20 | - |
| 3 | 40 | 40 | - | 20 | - |
| 4 | 60 | 40 | - | - | - |

North wins. Center is the Condorcet winner: nobody's first choice but everybody's second choice. But Center is eliminated in the first round of the Instant Runoff.

The good news about IRV is that second preferences still matter. Candidates representing small groups of voters, including small groups of centrists, are likely to be eliminated, but the voters who support them are still relevant. Supporters of small centrist parties are likely to be in a position to determine the outcomes of contests between larger parties. The election tends to go to whichever major party is more successful at wooing the center. Minor parties do no harm to major parties that resemble them and minor parties can hope to gradually grow and become major parties if they appeal to the center.

See Appendix A to see what happens if a minor party doesn't appeal to the center.

## Example set 2:

Monotonicity and runoff elimination order.
IRV can sometimes produce perverse results where a shift in support from candidate $X$ to $Y$ can cause $X$ to be eliminated instead of $Z$, which can result in $Z$ winning instead of $Y$.

Instant Runoff Voting ballots (2004 Canvass Dinner):

| Choice | Number of Voters |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | $\mathbf{5}$ | $\mathbf{2}$ | $\mathbf{8}$ | $\mathbf{3}$ | $\mathbf{3}$ |
| 1st | Chinese | Chinese | Italian | Mexican | Mexican |
| 2nd | Mexican | Italian | Mexican | Chinese | Italian |
| 3rd | Italian | Mexican | Chinese | Italian | Chinese |

Instant Runoff Voting

| Round | Chinese | Italian | Mexican |
| :---: | :---: | :---: | :---: |
| 1 | 7 | 8 | 6 |
| 2 | 10 | 11 | - |

Now suppose that the two voters who ranked Chinese and Italian respectively first and second changed their minds and decided to rank Italian first:

Instant Runoff Voting ballots (2005 Canvass Dinner):

| Choice | Number of Voters |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | $\mathbf{5}$ | $\mathbf{2}$ | $\mathbf{8}$ | $\mathbf{3}$ | $\mathbf{3}$ |
| 1st | Chinese | Italian | Italian | Mexican | Mexican |
| 2nd | Mexican | Chinese | Mexican | Chinese | Italian |
| 3rd | Italian | Mexican | Chinese | Italian | Chinese |

Instant Runoff Voting

| Round | Chinese | Italian | Mexican |
| :---: | :---: | :---: | :---: |
| 1 | 5 | 10 | 6 |
| 2 | - | 10 | 11 |

By voting sincerely, these two voters have caused the election to be
thrown from their first choice to their last choice. This is perverse, but it is a rather contrived example. It is also possible to come up with a scenario in which a small block of voters is large enough to determine whether or not an election using Condorcet's Method needs a circular tiebreaker. If they can predict the result of the tiebreaker, they get to choose whether they like the Condorcet winner or the (possibly different) tiebreaker winner better. This means that even Condorcet's Method is not absolutely immune from insincere voting. However, under Plurality, corresponding problems (ie. sensitivity to spoilers) are simply taken for granted.

## Example set 3:

Circular tiebreakers.

Condorcet ballots:

| Choice | Number of Voters |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
|  | $\mathbf{2}$ | $\mathbf{2}$ | $\mathbf{2}$ | $\mathbf{1}$ |
| 1st | Chinese | Italian | Mexican | Italian |
| 2nd | Italian | Mexican | Greek | Chinese |
| 3rd | Greek | Greek | Chinese | Greek |
| 4th | Mexican | Chinese | Italian | Mexican |

Condorcet Pairwise Results

| Chinese | beats | Italian | $4-3$ |
| :---: | :---: | :---: | :---: |
| Greek | beats | Chinese | $4-3$ |
| Mexican | beats | Chinese | $4-3$ |
| Italian | beats | Greek | $5-2$ |
| Italian | beats | Mexican | $5-2$ |
| Mexican | beats | Greek | $4-3$ |

Everyone has been beaten at least once. There is no Condorcet winner. We need a circular tiebreaker. Let's consider Nanson's Borda Elimination Method, Schulze's Beatpath Method, and Tideman's Ranked Pairs Method.

Tournament Graph (Directional or Digraph)


Nanson's Method
As a circular tiebreaker, let's try Nanson's "Borda Elimination" system first. An easy way to do this is to tabulate the votes in a matrix. Vij is the number of votes preferring candidate i over candidate j. Each pair of entries, Vij + Vji, should sum to the total number of voters. (If there are ballots that are not completely filled in, these sums will be wrong, and you need to add opposing pairs of half-votes to make sure that candidates are eliminated in the correct order.) Sum along rows to calculate the Borda Count. Eliminate the candidate with the lowest Borda Count by striking out the corresponding row and column. Repeat until there is one winner.

|  | Chinese | Italian | Greek | Mexican | Borda | Nanson | 2 Nanson 3 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Chinese | - | 4 | 3 | 3 | 10 | 7 | 4 |
| Italian | 3 | - | 5 | 5 | 13 | 8 | 3 |
| Greek | 4 | 2 | - | 3 | 9 | - | - |
| Mexican | 4 | 2 | 4 | - | 10 | 6 | - |

Chinese is the tiebreaker winner using Nanson's Method.

Now look at the beatpath method. A has a beatpath to $C$ if A beat $C$ or if $A$ beat $B$ and $B$ has a beatpath to $C$. The strength of a beatpath is the size of the smallest majority in the path. Pick the best path.

Figure 8
Beatpath Tournament Graph


| Candidates |  |  | Majority |
| :---: | :---: | :---: | :---: |
| Chinese | beats | Italian | 4 |
| Italian | beats | Chinese | 4 |
| Greek | beats | Chinese | 4 |
| Chinese | beats | Greek | 4 |
| Mexican | beats | Chinese | 4 |
| Chinese | beats | Mexican | 4 |
| Italian | beats | Greek | 5 |
| Greek | beats | Italian | 4 |
| Italian | beats | Mexican | 5 |
| Mexican | beats | Italian | 4 |
| dialect |  |  |  |
| Mexican | beats | Greek | 4 |
| dreekian |  |  |  |
| Greek | beats | Mexican | 4 |
| via Chinese |  |  |  |

Italian has beatpath wins against Greek and Mexican, eliminating them.

Chinese and Italian are tied as winners using the Beatpath Method. Now we need a conventional (pairwise) tiebreaker. I specified that we
would use the modified Bucklin's method for this, so we look at first place votes. Italian had 3 first place votes, and Chinese only had two. Italian therefore wins the tiebreaker.

Note for the very pedantic: If there are pairwise ties, technically you should distinguish between the Smith set and the Schwartz set (see Mike Ossipoff's web site).

The circular tiebreaker methods I describe here will automatically pick a winner who is a member of the Smith set, but you may be curious about other methods. To determine the Smith set, first find the Young winner, the candidate with the smallest number of pairwise defeats. There can be several tied (ie. Italian and Mexican). Include these in the Smith set. Recursively add any candidate who beats a member of the Smith set. All four are members in this example.

## Ranked Pairs

Now look at Tideman's "ranked pairs" method. Here we list the pairwise defeats in order from largest majority to smallest majority, "locking" any of these defeats that do not form cycles with the previously locked defeats, and discarding defeats that do.

| Candidates |  |  | Majority | Result |
| :---: | :---: | :---: | :---: | :---: |
| Italian | beats | Greek | 5 | locked |
| Italian | beats | Mexican | 5 | locked |
| Mexican | beats | Greek | 4 | locked using tiebreaker |
| Mexican | beats | Chinese | 4 | locked using tiebreaker |
| Chinese | beats | Italian | 4 | discarded |
| Greek | beats | Chinese | 4 | locked |

The largest majorities were Italian over Greek (5-2) and Italian over Mexican (5-2). The first two defeats can't cause any cycles, so we can lock them both without worrying about tiebreakers. The remaining defeats are tied in strength, all 4-3, but now we do need a tiebreaker. I specified earlier that we would use the modified Bucklin's method, so we look at first place support. Italian got 3 votes, Chinese got 2, Mexican 2, and Greek 0. This means that there is a 2-0 plurality supporting the Mexican win over Greek, a 2-2 draw regarding Mexican vs. Chinese, a 3-2 plurality that opposes the Chinese win over Italian, and a 2-0 plurality that opposes the Greek win over Chinese. The Mexican win over Greek thus has the most tiebreaker support, so we lock it. But this doesn't contradict any of the remaining defeats, so next we go to the Mexican win over Chinese and lock it. Now we discard the Chinese win over Italian because it would cause a cycle. Finally we lock the Greek win over Chinese, but the point is moot, because Italian has won the election.

## Example set 3, continued:

More fun with circular tiebreakers

Now let's go back and look at the guns and gays example from the section on implicit vs. explicit bargaining.

Voter preferences

| Number of voters | handgun ban | gay bashing |
| :---: | :---: | :---: |
| $35 \%$ | weakly support | strongly oppose |
| $40 \%$ | strongly oppose | weakly support |
| $25 \%$ | strongly support | weakly support |

Pairwise Results

| "gun ban" | beats | "gay bash" | with a $60 \%$ majority. |
| :---: | :---: | :---: | :---: | :---: |
| "both" | beats | "gun ban" | with a $65 \%$ majority. |
| "gun ban" | beats | "neither" | with a $60 \%$ majority. |
| "both" | beats | "gay bash" | with a $60 \%$ majority. |
| "gay bash" | beats | "neither" | with a $65 \%$ majority. |
| "neither" | beats | "both" | with a 75\% majority. |

Nanson's Method
First let's use Nanson's Method (Borda elimination). The gay bashing-only proposal gets eliminated in the first round, then "both," then "neither." The handgun ban-only proposal wins.


The gun ban passes using Nanson's Method, but the bill persecuting gays does not.

```
Marshall efficiency (net values of outcomes)
```

| Outcome | Group A | Group B | Group C | Value |
| :---: | :---: | :---: | :---: | :---: |
|  | $(35 \%)$ | $(40 \%)$ | $(25 \%)$ | (per 100 voters) |
| Gun Ban | $\$ 100$ | $-\$ 500$ | $\$ 100$ | $-\$ 14,000$ |
| Gay Bashing | $-\$ 1,000$ | $\$ 50$ | $\$ 50$ | $-\$ 31,750$ |
| Both | $-\$ 900$ | $-\$ 450$ | $\$ 150$ | $-\$ 45,750$ |
| Neither | $\$ 0$ | $\$ 0$ | $\$ 0$ | $\$ 0$ |

Figure 9
Tournament Graph (Condorcet)


| Candidates |  | Majority | Path |
| :--- | :--- | :---: | :---: |
| "gun ban" | beats | "gay bash" | $60 \%$ |
| direct |  |  |  |
| "gay bash" | beats | "gun ban" | $65 \%$ |
| "both" | beats | "gun ban" | $65 \%$ |
| "gun ban" | beats | "both" | diaither" and "both" |
| "gun ban" | beats | "neither" | $60 \%$ |
| "neither" | beats | "gun ban" | $65 \%$ |
| "both" | beats | "gay bash" | $60 \%$ |
| "gay bash" | beats | "both" | direct |
| "gay bash" | beats | "neither" | $65 \%$ |
| "neither" | beats | "gay bash" | $60 \%$ |
| viath" |  |  |  |
| "neither" | beats | "both" | direct |
| "both" | beats | "neither" | $60 \%$ |
| viarect |  |  |  |

"gay bash" has a beatpath victory over "gun ban", eliminating "gun ban".
"gay bash" has a beatpath victory over "both", eliminating "both". "gay bash" has a beatpath victory over "neither", eliminating "neither".

The "gay bash" proposal wins using the Beatpath method.

Ranked Pairs (Tideman)
Sort the victories by their decisiveness.

| Candidates |  | Majority | Result |  |
| :--- | :---: | :--- | :---: | :---: |
| "neither" | beats | "both" | $75 \%$ | locked |
| "both" | beats | "gun ban" | $65 \%$ | locked |
| "gay bash" | beats | "neither" | $65 \%$ | locked |
| "gun ban" | beats | "gay bash" | $60 \%$ | discarded |
| "gun ban" | beats | "neither" | $60 \%$ | discarded |
| "both" | beats | "gay bash" | $60 \%$ | discarded |

The "gay bash" proposal wins using the Tideman "Ranked Pairs" method.

## Peter's Report Card

In my opinion, the most important criterion for judging an election system is its vulnerability to manipulation. The worst such vulnerability is if it helps a candidate to have several second-rate clones of himself on the ballot. Borda suffers horribly from this problem, but some of the proposed circular tiebreakers for Condorcet are not entirely cloneproof either. (The ones I discuss are cloneproof if they are defined correctly.) Another fairly serious problem, also related to ballot access, is if spoilers are important, as they are under Plurality. Spoilers are a large part of the reason why Papua New Guinea's elections are prone to violence.

Another important criterion, or perhaps I should say family of criteria, is the degree to which a block of voters is rewarded for voting "strategically" rather than "sincerely." Again, the most disturbing example of this is the Borda count, where it is severely tempting to put my first choice first, his arch-rival last (even if he would honestly be my second choice), and the Monster Raving Loony candidate second. Borda is quite likely to reward voters for being downright malicious, injecting a huge amount of noise into the electoral signal. Plurality is less bad, rewarding me for betraying my favorite candidate in favor of the lessor of the two dominant evils. This also turns "front runner" status into a self-fulfilling prophecy, which is partly why money is so important in primary elections, and it makes it difficult to run serious reform candidates. Approval Voting, in comparison, is merely annoying. With Approval, I have to decide whether it's more important to support my first choice against my second choice, or my second choice against my last choice, because I can't do both. In order to make this decision, I need preelection public opinion poll information. The runoff systems, majority-runoff and Instant Runoff Voting), usually allow me to vote my conscience, but every once in a while the elimination order will get weird, and I will wish I had voted for the lessor of two dominant evils, as I would generally do under Plurality Voting. Condorcet (and most of the PR systems) allows me to vote my conscience with only very rare second thoughts. Under normal circumstances, predicting whether Condorcet will need a tiebreaker, and who is likely to win the tiebreaker, is too complicated and uncertain to give me a basis for strategic voting.

Another group of criteria, of moderate importance, have to do with the kinds of legislatures that result using these election systems. Is the resulting legislature likely to have a large enough number of centrist politicians that they will be able to play the extremists off against each other and form coalitions easily? According to this criterion, the centrists don't need to be a majority, but there need to be some. Another criterion is, does this legislature have the potential advantages associated with the implicit bargaining model? This criterion implies a relatively homogeneous legislature, with the centrists being a clear majority, but it potentially has the advantages of promoting the politics of persuasion rather than mobilization, and it also reduces the legislature's workload, because the most important compromises will have been determined on election day. Opposed to this is another criterion: does this legislature have the potential advantages associated with the explicit bargaining
model? These PR-style representatives seem to me to be more likely to be faithful to their constituents, and seem like a better solution to the "electoral straitjacket" problem. On the other hand, it may be argued that political fringe movements (ie. small parties) are full of fanatics, and that it is dangerous for fanatics to have "fair" representation.

There are also a number of less important criteria for voting systems. One of these is whether the ballots allow me to express my preferences accurately enough for the question of sincerity to be meaningful (which is the underlying problem with Approval Voting). Another, sharply conflicting criterion, is whether the complexity of the ballot becomes unmanageable with large numbers of candidates, as it tends to do with ranked ballots. Elections with large numbers of candidates are where Approval voting really shines.

| Keyword | Election System Judging Criteria |
| :--- | :--- |
| Clones | Do clones on the ballot help a candidate win? |
| Expressive | How fully can I express my preferences on the ballot? |
| Sincere | Do I have a strong incentive to misrepresent my <br> preferences? |
| Large \# | How well does the system deal with large numbers of <br> candidates? |
| Spoilers | Do similar candidates interfere with one another? |
| Stable | Is it hard for stable coalitions to form in the <br> legislature? |
| Implicit | Is much of the legislature's work taken care of on <br> election day? |
| Minorities | Are minority views represented well? |
| Fringes | Does it discourage "fringe" movements? |
| Overall | This is my utterly subjective overall opinion. |



| Plurality | pass | D | D | B | FAIL | pass | B | D | A | D | 1 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Majority-runoff | pass | C | B | B | pass | pass | B | D | A | D | 2 |
| Approval | pass | B | C | A | pass | pass | A | D | A | B |  |
| Borda Count | FAIL | A | F | C | pass | pass?? | C? | D | B | F | 3 |
| Instant Runoff | pass | A | B | C | pass | pass | A | D | A | B |  |
| Condorcet | pass | A | A | C | pass | pass | A | D | A | A | 4 |
| At Large | pass | D | D | B | FAIL | pass | B | F | A | F | 5 |
| Limited Vote | pass | C | D | B | FAIL? | pass? | C | C | C | D | 6 |
| Cumulative Vote | pass | B | D | B | FAIL? | pass?? | C | C | C | D | 6 |
| Closed Party List | pass | D | A | B | pass | FAIL | D | A | F | D |  |
| Open Party List | pass | C | B | B | pass | FAIL | D | A | F | D |  |
| Hybrid | pass | D | B | B | pass | pass | C | A | F | C |  |
| Single Trans. Vote | pass | A | A | D | pass | pass? | C | B | B | B |  |
| Loring's Method A | pass | A | A | D | pass | pass | C | B | B | B+ |  |

Comments:

1. Punishes third parties.
2. Expensive separate runoffs with voter turnout problems.
3. "...only for honest men."
4. Need circular tiebreaker.
5. Minority vote dilution issues.
6. Supports more than 2 parties, but they often still act as spoilers.

## Closing Questions

How much do primary elections matter?
Are the internal workings of political parties as important as the way the general elections work?

What are the effects of the differences between parliamentary and presidential systems?

What constitutes a "spoiled" ballot?
How do you handle incomplete rankings?
How do you break conventional ties, especially in elimination methods?

## Appendix A: Median Voter Model (Harold Hoteling)

Consider a society where high voter turnout can be taken for granted. (This is "the politics of persuasion" rather than "the politics of mobilization.") In this situation, political parties operating on a single-axis political spectrum under the two-party system are like two ice cream vendors on a long, narrow beach. Each one maximizes its success (votes, sales) by taking a position slightly toward the center from where the other (party, vendor) is stationed. In equilibrium, both of them are near the center. The winning political party is chosen by the median voter.


Minor extremist parties under Plurality Voting act as "spoilers," moving the effective median position further away from themselves. Presumably, voters tend to pick the candidate nearest to them on the left-right spectrum, but many will restrict themselves to one of the major parties. In this example, the Democrats could try moving to the left to attract Green voters, but in order to break even, they would have to attract two Green voters for every voter that they lost to the Republicans. (If voter turnout is an issue, the Greens are also likely to make it easier for the Republicans to mobilize their base.)


On the other hand, if runoff elections are held, small extremist
parties have little adverse effect. Their supporters revert to supporting the mainstream parties in the runoff election (MajorityRunoff and Instant Runoff Voting).


Mike Ossipoff objects that even with runoff elections, if an extremist party becomes larger than its more centrist sibling party, the centrist sibling party is eliminated from the runoff, and its supporters are split. Unless the extremist party rapidly assumes a centrist position, its behavior will again be counter to its expressed values.


Mike interprets this result as an argument in favor of either Approval Voting or Condorcet's Method. I interpret it as an indication that minor political parties make sense only if the political issue space is multidimensional.

## Appendix B: What constructive roles can third parties play in a twoparty system (Plurality Voting)?

This is a synopsis of http://www.ghg.net/redflame/spoilers.htm .

1. They can educate (indoctrinate? propagandize?) the voters, serving as publicity devices for philosophy or policy proposals (ie. the Socialist Party in the US a century ago).
2. They can try to replace an existing major party that has become too oligarchic and unresponsive to popular opinion (Abraham Lincoln's Republican Party).
3. They can deliberately run spoilers to try to blackmail the major parties into supporting electoral reforms such as Instant Runoff Voting (Britain's Liberal Alliance in the mid 1980s).
4. They can follow the Canadian model, where there is a two-party system in each province, but not the same two throughout the whole country. A new party could initially run candidates only in races or regions where one of the existing major parties is uncompetitive.
5. If political dividing lines are complicated, the two major parties may get confused (or indifferent) about where the center is. In this case, a minor candidate like George Wallace in 1968 can "help" them figure this out.

In practice, the Libertarian and Green parties in the US appear to me to be oriented more towards "self-expression" (see Maurice Duverger) than towards serious attempts to influence government policy. Neither of these parties has fully come to grips with the spoiler problem. I don't regard either of them as playing a generally constructive role.

## Appendix C: Block voting and the electoral college

See the Electoral College Primer, by Lawrence Longley and Neal Peirce, or my review of it, http://www.ghg.net/redflame/florida.htm .

The claim that the Electoral College is, on net, advantageous for the smaller states is false. The advantages the larger states get from block voting (the "unit rule") dwarf the advantages the smaller states get from having an extra two seats per state. Longley and Peirce refer to a statistical analysis that suggests that a voter from Montana is $3 / 5$ as valuable to a presidential candidate as a Floridian, and $3 / 8$ as valuable as a Californian. I made a simple statistical model of an imaginary country called "Slobovia" with an electoral college similar to ours and got similar results. The probability that a state's electors will be enough to tip the outcome in the electoral college is a highly nonlinear function of the size of the state. (See http://www.ghg.net/redflame/peter/slobovia.pdf .)

Why a voter's opinion should count for more or less depending on which state he lives in is frankly lost on me in the first place. Steven Hill's criticism of the Electoral College in Fixing Elections is exactly backwards (large vs. small states), but the bias strikes me as unjust regardless of who the beneficiaries are. Much of the sentiment opposed to abolishing the Electoral College appears to me to be little more than political ancestor worship.

Block voting is also bad for reasons that were discussed in the section on implicit vs. explicit bargaining.

Other criticisms of the Electoral College include the fact that a candidate can win without a plurality of the popular vote, and that winners often have a false mandate, with a very narrow plurality of the popular vote producing a "landslide" in the Electoral College.

A much more serious criticism of the Electoral College is that only some states are swing states, and political parties with presidential aspirations have enormous incentives to pander to swing voters in swing states at the expense of the rest of the country. Examples of this are the Clinton administration's handing of the Elian Gonzales incident and the Bush 43 administration's support for steel tariffs. The Electoral College allows swing voters in large swing states to "wag the dog."

Proponents of the Electoral College often claim that it gives a candidate an incentive to work towards uniform support throughout the country, working hardest for support in areas where he is weakest. In fact, neither candidate has an incentive to campaign hard in any area where they do not each already have close to $50 \%$ support. Neither Bush nor Gore campaigned seriously in Texas in 2000, despite Texas having the third largest number of Electors at 32. Fund raising yes, campaigning no.

The idea that the Electoral College helps prevent geographic polarization is even harder to take seriously if one considers minor candidates. A regional candidate like George Wallace can realistically hope to win some electoral college votes. A national candidate like Ross Perot cannot.

One defense of the Electoral College that I have not been able to refute is that it makes the election less sensitive to fraud. This is plausible, but I don't know how to model fraud. Andy Love argues that fraud is more likely in states where one party has an overwhelming advantage over the other, but that this sort of fraud is pointless given the Electoral College. Without the Electoral College, fraud would be a bigger problem. I don't see this as an insurmountable objection, but it needs to be addressed.

## Single Transferable Vote

Sample Ballot
You may vote in one of two ways:

## EITHER

By placing the single figure 1 in one and only one of these spaces to indicate the voting ticket (preference schedule) you wish to adopt as your vote

| Fish Party | Reptile Party $\quad$ Bird Party | Mammal Party |  |
| :---: | :---: | :---: | :---: | :---: | :---: |

OR
By placing the numbers 1 to 19 in the order of your preference

| Fish Party | Reptile Party | Bird Party | Mammal Party | independent |
| :---: | :---: | :---: | :---: | :---: |
| Karl Kipper | Bret Boa | Mike Mallard | Sue Squirrel | Anne Amoeba |
| Tom Tuna | Gwen Gecko | Wayne Wren | Fred Ferret | Larry Lobster |
| Cathy Chinook | Inge Iguana | Jane Jacana | Omar Ocelot |  |
| Dan Dolphin |  | Ned Nuthatch | Paul Poodle |  |
| Emily Eel |  |  | Hank Hamster |  |

The STV example ballot in the I-IDEA Handbook is from the 1996 Victoria election to the Australian Senate. This ballot has 11 named parties, one unnamed group, and 6 independents, totalling 44 candidates, competing for 6 seats. I can't read the as-reproduced instructions without a magnifying glass.

## Addendix D: Vote processing using Single Transferable Vote (STV).

STV is like Instant Runoff Voting (IRV), except that there are more than one seat per district to be won. Each ballot is assigned to the voter's most preferred candidate. The number of votes needed to win a seat is called a "quota." Candidates who reach the quota are seated and their excess votes are redistributed. When no remaining candidate has a quota, the weakest one is eliminated, as with IRV, and again votes are redistributed. The process ends when all of the seats have been filled or when the number of candidates remaining is equal to the number of unfilled seats. With votes assigned in this way, parties tend to receive roughly proportional representation, especially if the number of seats per district is large (ie. 5 rather than 2), so this is considered a PR system. STV is the system used to elect the Australian Senate.

But how do you decide which votes are to be used to elect a candidate, and which are excess, to be redistributed? Given computers, and the need to make the vote counting process reproducible, an obvious way to do this is to give each vote an initial weight of 1.0 , and pro-rate the remaining weight of each vote according to what fraction of a candidate's votes were needed to make quota. This is called Fractional STV.

Here are the voters' preference schedules from a committee election that used Fractional STV, with 25 voters, 9 candidates, and 2 seats:

STV Raw Voting Results

|  | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 | 24 | 25 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| A | 5 | 5 | 8 | 9 | 1 | 6 |  | 5 |  |  | 2 | 3 | 5 | 3 | 3 | 2 | 3 |  | 1 | 9 | 2 | 6 | 1 |  | 8 |
| B | 3 | 4 | 4 | 4 | 5 | 1 | 4 | 6 | 1 |  | 3 | 8 | 3 | 9 | 5 | 4 | 6 |  | 2 | 7 | 3 | 7 | 6 |  | 5 |
| C | 6 | 6 | 6 | 8 | 4 | 4 |  | 7 | 5 |  | 5 | 6 | 6 | 8 | 4 | 9 | 5 |  | 6 | 5 | 4 | 8 | 9 | 3 | 3 |
| D | 7 | 7 | 9 | 5 | 9 | 8 |  | 8 |  |  | 1 | 9 | 8 | 6 | 7 | 7 | 4 | 2 | 8 | 6 | 5 | 9 | 3 |  | 9 |
| E | 1 | 1 | 1 | 1 | 2 | 3 | 1 | 1 | 4 | 3 | 6 | 5 | 1 | 5 | 2 | 5 | 2 | 1 | 5 | 2 | 9 | 1 | 4 | 4 | 1 |
| F | 9 | 3 | 7 | 7 | 3 | 5 |  | 2 |  |  | 7 | 1 | 7 | 7 | 8 | 8 | 7 |  | 7 | 4 | 6 | 2 | 7 |  | 6 |
| G | 8 | 2 | 2 | 3 | 8 | 9 | 3 | 4 | 2 |  | 9 | 2 | 2 | 2 | 1 | 1 | 8 |  | 4 | 3 | 7 | 3 | 8 | 2 | 7 |
| H | 2 | 9 | 3 | 2 | 6 | 7 | 2 | 3 |  |  | 8 | 7 | 9 | 4 | 6 | 6 | 1 | 3 | 9 | 1 | 1 | 4 | 2 | 1 | 4 |
| I | 4 | 8 | 5 | 6 | 7 | 2 |  | 9 | 3 |  | 4 | 4 | 4 | 1 | 9 | 3 | 9 |  | 3 | 8 | 8 | 5 | 5 |  | 2 |

Note that several voters did not completely fill in their rankings. In Australia, ballots with incomplete rankings are considered "spoiled," but ranking a long list of candidates quickly becomes tedious, so each party on the ballot has a box that the voter can check in order to fill in the rankings according to that party's recommendation. However, in this committee election, incomplete
ballots were considered valid. The disadvantage of allowing incomplete rankings is that if all the candidates who were listed on a ballot get eliminated or seated, the remainder of that ballot's weight is wasted. If many ballots are incomplete, some candidates may win seats with less than a full quota of votes.

Another complication is skipped rankings. Look closely at ballot \# 10. How do you interpret this ballot? In this election, the skipped rankings were ignored, and the preferences were shifted downward to fill in the gaps. Thus the 3rd choice on ballot \# 10 is interpreted as a lst choice. In some cases, it may not be obvious if this is really what the voter intended. The rules for handing strangely marked ballots need to be specified in advance.

Now we have to define a quota. There is a range of possible values that make sense. The "Hare quota" is the number of votes divided by the number of seats, rounded down to the nearest integer. With fractional STV, there is no reason why a quota has to be an integer, so we could just leave this as a real number. This would be the upper limit on a reasonable size for a quota. A "Droop quota" would be the number of votes divided by one more than the number of seats, rounded up so that it isn't quite possible for more candidates to reach quota than there are seats to win. Again, with fractional STV, this doesn't have to be an integer, so instead of rounding up, we could leave it as a real number and add a very small increment, such as 0.000001 . This would be the lower limit on a reasonable size for a quota. The Hare quota minimizes the number of votes that are "wasted" because they are not assigned to a winning candidate, but this is not as good as it sounds because a vote might end up being assigned to a candidate who was closer to the bottom than to the top of the voter's list of preferences. The Hare quota also is most affected by incomplete ballots. With a high quota, votes with incomplete rankings are more likely to be wasted because all of the ranked candidates were eliminated. This makes it more likely that thinly supported candidates will win seats towards the end of the process (with significantly less that a full quota). For this election, we used the non-integer Droop quota (with a 0.000001 offset). Note that how a quota is defined can determine who wins a seat, so it needs to be specified in advance.

Note that if you increase the number of seats to be won from 2 to 3, you can't guarantee that the people who would have won the 2 seats will be among the winning 3.

Since this is a committee election, we are likely to need tiebreakers, which need to be specified in advance. This is complicated by the fact that we are using real numbers rather than integers, and so we need to worry about round-off error. We may also need to worry about round-off error in determining whether or not a candidate makes quota. The first tiebreaker used for this election was the amount of support the tied candidates had in previous rounds, starting with the most recent. This requires some additional bookkeeping. The second tiebreaker is what total support would have been if all the other, non-tied, candidates were eliminated.

Now we are ready to process the ballots.
First we calculate the quota. This comes to $81 / 3$ votes.

## Initial Support

| Candidate | A | B | C | D | E | F | G | H | I |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| support | 3.000 | 2.000 | 0.000 | 1.000 | 11.000 | 1.000 | 2.000 | 4.000 | 1.000 |

Next, we check whether any candidates have already made quota. Yes, Candidate E has 11 votes. Candidate E is now seated, and removed from the list of active or "continuing" candidates. Also, 8.333/11 of the votes that elected E have been used up, so we multiply the weighting factors of these votes by (2.667/11). Next we redistribute these votes to whichever candidates are ranked next. Recall that voter \# 10 only listed one candidate on his ballot. The remaining weight of ballot \# 10 is thus wasted.

| Candidate | A | B | C | D | E | F | G | H | I |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| support | 3.000 | 2.000 | 0.000 | 1.242 | WIN | 1.485 | 2.727 | 4.727 | 1.242 |

No one else has a quota, even after redistributing the remains of E's votes, so now we start eliminating candidates. The first to go is Candidate C, who has 0 votes. There is nothing to redistribute, so we continue eliminating.

Now we have a tie between Candidates D and I, with 1.242 votes each. The first tiebreaker doesn't resolve the tie because both candidates had 1 vote in each of the previous steps. The second tiebreaker is who would have won if only the tied candidates were in the race. D is preferred by voters 2, 4, 8, 11, 15, 17, 18, 20, 21 and 23 (10 votes). I is preferred by voters 1, 3, 5, 6, 9, 12, 13, 14, 16, 19, 22, and 25 (12 votes). D is thus eliminated, and his votes redistributed.

| Candidate | A | B | C | D | E | F | G | H | I |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| support | 4.000 | 2.000 | --- | --- | WIN | 1.485 | 2.727 | 4.970 | 1.242 |

Candidate I is eliminated next, and his votes redistributed.

| Candidate | A | B | C | D | E | F | G | H | I |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| support | 4.000 | 2.000 | --- | --- | WIN | 1.485 | 3.727 | 5.212 | --- |

Now we eliminate Candidate $F$ and redistribute his votes. The order of elimination of $D$ and $I$ didn't matter after all.

| Candidate | A | B | C | D | E | F | G | H | I |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| support | 4.000 | 2.000 | --- | --- | WIN | --- | 4.970 | 5.455 | --- |

Candidate B is eliminated next, and his votes redistributed.

| Candidate | A | B | C | D | E | F | G | H | I |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| support | 5.000 | --- | --- | --- | WIN | --- | 5.970 | 5.455 | --- |

Candidate A is eliminated next, and his votes redistributed.

| Candidate | A | B | C | D | E | F | G | H | I |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| support | --- | --- | --- | --- | WIN | --- | 6.970 | 9.455 | --- |

Candidate $H$ has reached quota and is awarded the second seat. The election is over.

I also printed out some statistics out of curiousity:
The total percentage of wasted vote weight was 33.3333253
The total percentage vote loss due to incomplete ballots was 1.0846925
The total number of completely wasted votes was 6 The number of votes completely lost due to truncated rankings was 0

I will be happy to send a copy of the Pascal source code I used to process this data to anyone who is interested. Email me at murmur@ghg. net .

## Appendix E: The Paradox of Voting.

Given the number of people who vote, the probability of a major election coming down to one vote is less than the probability of being hit by lightning on the way to the polling place (see Mueller, p. 350). So why do people vote in the numbers that they do?

One answer is that voters are ignorant of the odds. But this is strange, considering that better educated people are more likely to vote than less educated people. Also, I don't have to know what the odds are in order to know that they are vanishingly small and that my time is valuable.

Another answer is that voters are "irrational." Their behavior either is not goal-oriented, or they are too stupid or ignorant to know how to achieve their goals.

A third answer is that many people would rather accept some probability of immediate death than accept an equal probability of an opposing politician being elected. This is not plausible in any even remotely sane country, where the government is led by parties or coalitions of parties that are at least somewhat centrist.

A fourth answer is that voters are rational, but their reasons for voting are subtle, and do not depend strongly on the possibility of the election coming down to one vote. For example, they may be responding to peer pressure. Or they may be indulging in a bit of pleasant fantasy (like buying a lottery ticket).

This fourth explanation is my favorite, but it suggests that it will be hard to come up with a realistic or reliable model of how voters behave. How can I predict behavior that may be largely motivated by fantasy? We must be careful not to take too seriously any argument that assumes that voters are "rational" in any obvious sense. When I know that the odds against an election coming down to one vote are astronomical, $I$ have very little incentive to be honest with myself about my motives. I may pretend to be more altruistic than I really am. Or I may pretend that the people on the other side of the railroad tracks aren't fully human and don't have human rights.

Speaking of the irrationality of everyday political behavior, I highly recommend John McCarthy's essay on "ideological tribalism." He views the left-right spectrum as the result of political alliances and non-critical thinking rather than philosophical coherence.
http://www-formal.stanford.edu/jmc/progress/ideology.html
There is also a "soccer hooliganism" model described in David D. Friedman's book, Hidden Order, that views a great deal of political behavior in terms of sports fandom. John Cleese and Robin Skynner develop the same point in their discussion of "paranoia" in Families and How to Survive Them.

## References (in no particular order)

For information about electoral reform in general, visit The Center for Voting and Democracy (CVD), http://www.fairvote.org/ .

I strongly recommend George Hallet's critique of the Objections to PR ( http://www.mtholyoke.edu/acad/polit/damy/hallet.htm ) on Douglas Amy's PR Library website ( http://www.mtholyoke.edu/acad/polit/damy/prlib.htm ).

Reflecting All of Us: The Case for Proportional Representation , Robert Richie and Steven Hill, ed., Beacon Press, Boston, 1999, ISBN 0807044210 . This book is notable for the informed skepticism of the chapters by Ferejohn and Cox ( http://bostonreview.mit.edu/BR23.1/ ).

Real Choices/New Voices: The Case for Proportional Representation in the United States, Douglas J. Amy, Columbia University Press, New York, 1993, ISBN 0-231-08154-5. This book is about PR, and says little about single seat reforms such as IRV.
"PR: The Case for a Better Election System," Douglas J. Amy (available from CVD).

Also available through CVD is The International IDEA Handbook of Electoral System Design, published by the Institute for Democracy and Electoral Assistance, with many good case studies, most notably the one on Papua New Guinea. Highly recommended! http://www.int-idea.se/publications/system-design.html

Approval Vote web site. http://bcn.boulder.co.us/government/approvalvote/center.html

Here's another.
http://www-personal.umich.edu/~bpl/approval-vote.htm
And here's one with some nice graphics. http://www.tursiops.cc/idhop/av/

Rob Lanphier's site has links to some hard-core technical information and debate, including a Perl script. http://www.eskimo.com/~robla/politics

Mike Ossipoff's site has more hard-core technical discussion. He also likes Approval Voting.
http://www.barnsdle.demon.co.uk/vote/vote.html
The people at Election Methods also like Approval Voting. They offer a Python script for Condorcet. http://www.electionmethods.org

Robert LeGrand's descriptions of ranked-ballot voting methods has detailed examples of how many of these systems differ. This is also hard-core. http://userfs.cec.wustl.edu/~rhl1/rbvote/desc.html

Robert Loring's site discusses a number of election systems, most notably his Ensemble Rule A (LERa). Very readable. Has several free software downloads.
http://accuratedemocracy.com/elect.html
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See Is Democracy Fair: The Mathematics of Voting and Apportionment by Leslie Johnson Nielsen and Michael de Villiers, Key Curriculum Press, 1997, ISBN 1559532777 for very readable discussions of Arrow's Impossibility Theorem, Nanson's Method, and voting issues in general. It is an example-oriented high school curriculum book, with examples involving things like camping trips and meat vs. vegetarian pizzas. http://www.keypress.com
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See The New World of Economics, 3rd ed., by Richard McKenzie and Gordon Tullock, Irwin, Inc., 1981, ISBN 0256024944 for a discussion of the "median voter model."

See The Calculus of Consent: Logical Foundations of Constitutional Democracy, by James M. Buchanan and Gordon Tullock, University of Michigan Press, 1962, ISBN 0472061003 for a discussion of explicit vs. implicit bargaining.

See Public Choice II, by Dennis Mueller, Cambridge University Press, 1989, ISBN 0521379520 for a theoretical discussion of the role of the "agenda setter." This is a very dense, academic economics book, and discusses some sophisticated voting systems suitable for use within legislatures and professional committees. However, Mueller's discussion of the Borda count seems strangely naive after reading Nielsen's and de Villiers' high school curriculum.

